CHAPTER 01

FUNCTIONS AND LIMITS

BASIC DEFINITIONS AND FORMULAS

Open Interval: The set of all points x such that x lies between two real numbers a & b (a < b) so that a < x < b is called the Open Interval, denoted by (a, b) or] a, b [Hence (a, b) = {x / a < x < b}

Closed Interval: The set of all points x such that x lies between two real numbers a & b (a < b) So that a ≤ x ≤ b is called the Closed Interval denoted by [a , b] Hence [a, b] = {x / a ≤ x ≤ b}

Lower bound: Let $\phi \neq S \subset R$ (i.e. S is a non-empty subset of R). A number $a \in R$ is called a Lower bound of S. if $a \leq x, \forall x \in S$.

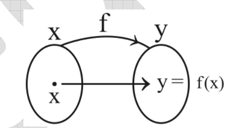
Greatest lower bound (g.l.b.): Let $\phi \neq S \subset R$, Let a_1,a_2,a_3 …..a_n….. are lower bonds of S. If “m” is the smallest of all these lower bounds $a_1a_2a_3$ …..a_n….. then “m” is called greatest lower bound (g.l.b) of S.

Upper Bound: Let $\phi \neq S \subset S$, A number $b \in R$ is called on upper bound of S if $b \geq x, \forall x \in S$.

Least Upper Bound (l.u.b.): Let $\phi \neq S \subset R$, Let b_1,b_2,b_3 …..b_n….. are all upper bounds of S. If “M” is the least of all these upper bounds then “M” is called least upper bound (l.u.b.)

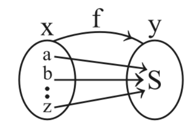
Function: Let x and y be two non-empty Sets. A rule f which assigns to each elements of x a unique element of y is called a function or (single – valued) mapping (or map) from x to y. If $x \in x$ then the element y of y assigned to x, under the rule f is called the image of x, or the value f at the point x. If y is the image of x, we write y = f(x).

If f is a function from x to y, then the set x is called the domain of the function and the subset of y consisting of all the images is called its range.



Types of Functions

1. Constant Function: A function $f : x \to y$ is called a constant function if the range of f is a single element.



Since Range of f = {S} ∴ f is a constant function.

2. Identify Function: A function $f : x \to x$ is called Identify Function.

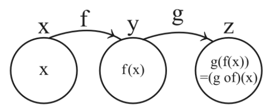
3. One-One (or Injective) Function: A function $f : x \to y$ is called one-one (or injective) if distinct elements of x have distinct images in y i.e. whenever $f(x_1) = f(x_2)$ $\forall x_1,x_2 \in x$ then $x_1 = x_2$.

4. Onto (or Surjective) Function: A function $f : x \to y$ is called onto (or Surjective) if each element of y is the range of x i.e. range of f = y.

5. One-One and Onto Both (or Bijective) Function: A function f: $x \to y$ is called bijective if it is both one-one (or injective) and onto (or Surjective)

6. Inverse Function: A function $f^{-1}: y \to x$ is called inverse of f: $x \to y$.

7. Composition of Functions: Let $f : x \to y$ and $g : y \to z$ be any two functions. The composition (or resultant) g of these functions.



is definition to be the function from x to z, given by (g of) (x) = g (f(x))

8. Even Function: A function $f : x \to R$, where $x \subset R$ is called even if $f(-x) = f(x) \;\forall\; x \in x$.

9. Odd Function: A function $f : x \to R$, where $x \subset R$ is called odd if $f(-x) = -f(x) \;\forall\; x \in x$.

(1) Sequence: A group of numbers arranged according to certain rule is called a Sequence.

(2) Limit of a Sequence: Consider the Sequence $\left\{1, \frac{1}{2}, \frac{1}{3} \ldots \frac{1}{n}, \ldots\right\}$ we depiet the terms of this sequence on real line. We see that all the terms of the sequence tend to zero and no term of the sequence will be beyond zero. Hence we say that zero is the Limit or Limit pont of the sequence.

(3) $\lim\limits_{n\to\infty} n = \infty$

(4) $\lim\limits_{n\to\infty} \frac{1}{n} = 0$

(5) $\lim\limits_{n\to\infty} (n \pm a) = \infty$

(6) $\lim\limits_{n\to\infty} \frac{1}{n \pm a} = 0$

(7) $\lim\limits_{n\to\infty} \left(1 + \frac{a}{n}\right)^{bn} = e^{ab}$

(8) $\lim\limits_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} = e$

(9)Monotonic Sequence Increasing: A sequence $\{a_n\}$ is called monotonic increasing if $a_1 \le a_2 \le a_3 \le a_4 \le \ldots \le a_n \le a_{n+1} \le \ldots$..

(9)Monotonic Sequence Decreasing: A sequence $\{a_n\}$ is called monotonic decreasing if $a_1 \ge a_2 \ge a_3 \ge a_4 \ge \ldots \ge a_n \ge a_{n+1} \ge \ldots$.

(10) Convergent Sequence: If $\lim\limits_{n\to\infty} a_n \ne \infty$

(11) Divergent Sequence: If $\lim\limits_{n\to\infty} a_n = \infty$

(12) Arithmetic Sequence: General term $a_n = a + (n-1)\, d$, where $d = T_2 - T_1$

(13) Geometric Sequence: General term $a_n = ar^{n-1}$, $r = \frac{T_2}{T_1}$

(14) Arithmetic Series: $S_n = \frac{n}{2}\{2a + (n-1)\, d\}$

(15) Geometric Series: $S_n = \frac{a\,(r^n - 1)}{r - 1}$, $r > 1$

$$S_n = \frac{a\,(1 - r^n)}{1 - r}, \; r < 1$$

(16) $\ln(AB) = \ln A + \ln B$

(17) $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

(18) $\ln A^m = m \ln A$

(19) $a^x = e^{x \ln a}$

Sum the series

(20) $x_1 + x_2 + x_3 + \ldots\ldots + x_n = \sum\limits_{i=1}^{n} x_i$

(1) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

(2) $\lim\limits_{x\to\infty} \left(1 + \frac{1}{x}\right)^{x} = e$

(3) $\lim\limits_{x\to\infty} (1+3)^{\frac{1}{x}} = e$

(4) $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$

(5) $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

(6) $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

(7) $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$

(8) $\lim_{x\to\infty} \frac{\ln(1+e^x)}{x} = 1$

(9) Continuity of a function: Let f be a function defined on a neighbourhood of a $\in$ R. Then f is continuous at a if and only if

$$\lim_{x\to a} f(x) = f(a)$$

(10) Binomial Theorem:

(i) $(a + b)^n = a^n + {}^nc_1\, a^{n-1}b + {}^nc_2\, a^{n-2}b^2 + \text{-----} + b^n$

(ii) $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \text{-----}$

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CHAPTER 02

THE STRAIGHT LINE

BASIC DEFINITIONS AND FORMULAS

Definition of Isosceles triangle: If two sides are equal in length is called isosceles Δ.

Definition of Equilateral triangle: If three side are equal in length is called equilateral Δ.

Definition of Right angled triangle: If one angle is right (90^o) is called right angled Δ also satisfied Pythagoras theorem ($H^2=B^2+P^2$)

Definition of Circle: A circle is a set of points in R $\times$ R whose distance from a given fixed point remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

Definition of Parallelogram: The opposite sides and angles are equal (same) is called a Parallelogram.

Definitoin of Square: The all sides are equal in length and all angled are 90^o is called a square.

Definition of centroid: The point of concurrence of median is called centroid "G".

Definition of Incentre: Centre of the inscribed circle is called incentre "I" which is meeting point of internal bisectors of the triangle.

Internal bisector: A line which bisects the angle at any ratio at the same ratio, it divides the opposite side of that angle of the triangle.

Definition of orthocentre: The point of concurrence of attitudes is called an another.

Definition of Slope of a line or Gradient of a line: The trigonometric tangent of the inclination is called the slope (or gradient) of the line and is denoted by m thus m = tanθ.

Concurrency of three lines: Three lines L_1, L_2 & L_3 are said to be concurrent, if they interest at only one point and the point "O" where the three lines interest is called point of concurrency.

(1) DISTANCE FORMULA IN CARTESIAN COORDINATES:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(2) DISTANCE FORMULA IN POLAR COORDINATES:

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\ \text{Cos}(\theta_2 - \theta_1)}$$

(3) MID-POINT FORMULA:

$$x = \frac{x_1 + x_2}{2},\ \ y = \frac{y_1 + y_2}{2}$$

(4) INTERNAL DIVISION FORMULA, ID (x, y):

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2},\ \ y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

(5) EXTERNAL DIVISION FORMULA, ED (x, y):

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2},\ \ y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$

(6) CENTROID, G (x, y):

$$x = \frac{x_1 + x_2 + x_3}{3},\ \ y = \frac{y_1 + y_2 + y_3}{3}$$

(7) IN-CENTRE, I(x, y):

$$x = \frac{ax_1 + bx_2 + cx_3}{a + b + c},\ \ y = \frac{ay_1 + by_2 + cy_3}{a + b + c}$$

(8) SLOPE OF A LINE (OR GRADIENT OF A LINE):

$$\text{Slope} = m = \tan\theta,\ m = \frac{y_2 - y_1}{x_2 - x_1}$$

(9) CONDITION OF THREE COOLINEAR POINTS
A (x_1, y_1), B (x_2, y_2), C (x_3, y_3),:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$

(10) PARALLEL LINES:

$$m_1 = m_2$$

(11) PERPENDICULAR LINES:

$$m_1 \times m_2 = -1$$

(12) ANGLE BETWEEN TWO LINES:

$$\text{Tan}\theta = \frac{m_2 - m_1}{1 + m_2m_1}$$

EQUATIONS OF STRAIGHT LINES

(13) LINE PARALLEL TO X – AXIS:

$$y = b$$

(14) LINE PARALLEL TO Y – AXIS:

$$x = a$$

(15) POINT SLOPE FORM:

$$y - y_1 = m_1 (x - x_1)$$

(16) TWO POINTS FORM:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \quad \text{OR} \quad \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(17) SLOPE-INTERCEPT FORM:

$$y = mx + b$$

(18) TWO INTERCEPTS FORM:

$$\frac{x}{a} + \frac{y}{b} = 1$$

(19) PERPENDICULAR FORM:

$$x \cos\alpha + y \sin\alpha = P$$

(20) SYMMETRIC FORM:

$$\frac{x - x_1}{\text{Cos}\theta} = \frac{y - y_1}{\text{Sin}\theta}$$

CHAPTER 03

THE GENERAL EQUATIONS OF STRAIGHT LINES

BASIC DEFINITIONS AND FORMULAS

(1) General Form of Straight line.

$Ax + By + C = 0$

(2) Slope of line = m = $\dfrac{-\text{Coefficient of } x}{\text{Coefficient of } y} = \dfrac{-A}{B}$

(3) N.B. $L_1 \equiv a_1x + b_1y + c_1 = 0$

$L_2 = a_2x + b_2y + c_2 = 0$ are two straight line

Then they (i) Intersect if $a_1 b_2 - a_2 b_1 \neq 0$

(ii) Parallel if $a_1 b_2 - a_2 b_1 = 0$

(iii) Perpendicular if $a_1 a_1 + b_1 b_2 = 0$

(iv) Concident if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

(v) Parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

(4) Angle between L_1 and L_2

$$\text{Tan}\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

(5) Point of intersection between L_1 and L_2

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

(6) Condition of concurrency for three lines.

$L_1 \equiv a_1x + b_1y + c_1 = 0$

$L_2 \equiv a_2x + b_2y + c_2 = 0$

$L_3 \equiv a_3x + b_3y + c_2 = 0$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

(7) Straight line through the intersection of two given lines

$L_1 + KL_2 = 0$

(8) Equation of the bisectors of angles between two straight lines

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

(9) Area if a triangle = Δ

$$\Delta = \frac{1}{2} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(10) Condition for three collinear points then $\Delta = 0$

(11) Area of a quadrilateral

Area = (base) (attitude)

$$\text{Area} = \frac{1}{2} = \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

(12) Area of a square or Rectangle

Area = Length $\times$ breadth

(13) Equation of a pair of lines through the origin

$ax^2 + 2hxy + by^2 = 0$

(14) Nature of Rools If $b \neq 0$ then

$ax^2 + 2hxy + by^2 = (y - m_1x)(y - m_2x) = 0$

Where $m_1 = \dfrac{-h + \sqrt{h^2 - ab}}{b}$ and $m_2 = \dfrac{-h - \sqrt{h^2 - ab}}{b}$

$h^2 - ab = D$ is called the discriminant.

(i) If $D = h^2 - ab > 0$ then two real and distinct lines.

(ii) If $D = h^2 - ab = 0$ then two real and coincident lines.

(iii) If $D = h^2 - ab < 0$ then two imaginary and distinct lines.

(15) Angle between the lines $ax^2 + 2hxy + by^2 = 0$ then

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$ (i) $h^2 - ab = 0$ lines are coincident.

(ii) $a + b = 0$ lines are perpendicular.

(16) Combined equation of the two straight lines which are perpendicular to the given pair of lines $x^2+(m_1+m_2)\,xy+m_1 \bullet m_2\, y^2 = 0$

(17) Distance Formula point to line:

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

(18) Area of a $\Delta = r_1r_2 \text{ Sin } (\theta_2 - \theta_1) + r_2r_3 \text{ Sin } (\theta_3 - \theta_2) + r_3r_1 \text{ Sin } (\theta_1 - \theta_3)$

CHAPTER 04

DIFFERENTIABILITY

BASIC DEFINITIONS AND FORMULAS

(1) Derivative Or Differentiation: The rate of change of dependent variable with respect to independent variable is called derivative or differentiation.

(2) Derivative by First Principle:

(i) $f'(a) = \lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$

(ii) $f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Derivatives

(3) $\frac{d(c)}{dx} = 0$

(4) $\frac{d(x^n)}{dx} = nx^{n-1}$

(5) $\frac{d}{dx}(ax+b)^n = na\,(ax + b)^{n-1}$

(6) $\frac{d}{dx}(e^x) = e^x$

(7) $\frac{d}{dx}(e^{ax}) = ae^{ax}$

(8) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

(9) $\frac{d}{dx}\ln(ax+b) = \frac{a}{ax+b}$

(10) $\frac{d}{dx}(\sin x) = \cos x$

(11) $\frac{d}{dx}(\cos x) = -\sin x$

(12) $\frac{d}{dx}(\tan x) = \sec^2 x$

(13) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(14) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(15) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

(16) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(17) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(18) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(19) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

(20) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(21) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

(22) $\frac{d}{dx}(UV) = U\left(\frac{dv}{dx}\right) + V\left(\frac{du}{dx}\right)$

(23) $\frac{d}{dx}(UVW) = UV\left(\frac{dw}{dx}\right) + UW\left(\frac{dv}{dx}\right) + VW\left(\frac{du}{dx}\right)$

(24) $\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\left(\frac{du}{dx}\right) - U\left(\frac{dv}{dx}\right)}{V^2}$

(25) $\frac{d}{dx}(a^x) = a^x \ln a$

(26) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ (Parametric Differertiation)

CHAPTER 05

APPLICATION OF DIFFER-ENTIAL CALCULUS

BASIC DEFINITIONS AND FORMULAS

(1) Geometrical Meaning of $\frac{dy}{dx}$ is equal slope of tangent.

(2) Rate of change of y with respect to x is equal to $\frac{dy}{dx}$.

(3) Speed = $\frac{ds}{dt}$

(4) Magnitude of Acceleration = $\frac{d^2s}{dt^2}$.

(5) Volume of Sphere = V = $\frac{4}{3}\pi r^3$.

(6) Volume of Right Circuler Cone = V = $\frac{1}{3}\pi r^2 h$.

(7) Approximate value = y + Δy where $\Delta y = \frac{dy}{dx} . \Delta x$

(8) Approximate value of Sin $(x + \Delta x) = \sin x - \cos x \ \Delta x$

(9) Approximate value of Cos $(x + \Delta x) = \cos x - \sin x \ \Delta x$

(10) Approximate value of tan $(x + \Delta x) = \tan x + \sec^2 x . \Delta x$

(11) Approximate value of $\log_{10}(x + \Delta x) = \log_{10} x + \frac{\log_{10} e}{x} \Delta x$

(12) Approximate value of $\sqrt{x + \Delta x} = \sqrt{x} + \frac{1}{2\sqrt{x}} . \Delta x$

(13) Extreme Values

f (x) = Given Function

f′ (x) = 0 (tangent of curve parallel to x–axis)

$f''(x_1) = < 0 \Rightarrow x_1$ is Maximum

$f''(x_2) = > 0 \Rightarrow x_2$ is Minimum

$f''(x_3) = 0 \Rightarrow x_3$ is neither maximum nor minimum

(14) Volume of Cube = V = Length × breadth × height

CHAPTER 06

ANTIDERIVATIVES

BASIC DEFINITIONS AND FORMULAS

DefinitionI: The reverse process of derivative is called antiderivative or integration.

Standard formulas for Integration.

(1) $\int dx = x + c$

(2) $\int x\, dx = \frac{x^2}{2} + c$

(3) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$

(4) $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

(5) $\int \frac{1}{x} dx = \ln x + c$

(6) $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln (ax+b) + c$

(7) $\int e^x\, dx = e^x + c$

(8) $\int a^x\, dx = \frac{a^x}{\ln a} + c$

(9) $\int e^{ax}\, dx = \frac{e^{ax}}{a} + c$

(10) $\int \sin x\, dx = -\cos + c$

(11) $\int \cos x\, dx = \sin x + c$

(12) $\int \tan x\, dx = \ln \sec x + c$

(13) $\int \sec x\, dx = \ln (\sec x + \tan x) + c$

$= \ln \tan \left(\frac{x}{2} + \frac{\pi}{4}\right)$

(14) $\int \sec^2 x = \tan x + c$

(15) $\int \sec x \tan x\, dx = \sec x + c$

(16) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(17) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(18) $\int \cot x\, dx = \ln \sin x + c$

(19) $\int \operatorname{cosec} x\, dx = \ln (\operatorname{cosec} x - \cot x) + c$

$= \ln \tan \left(\frac{x}{2}\right) + c$

(20) $\int \sec^3 x\, dx = \frac{1}{2} \sec\theta \tan\theta + \frac{1}{2} \ln (\sec\theta + \tan\theta) + c$

(21) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

(22) $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

(23) $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + c$

(24) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left(\frac{x-a}{x+a}\right) + c$

(25) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln \left(\frac{a+x}{a-x}\right) + c$

(26) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln (x + \sqrt{x^2 + a^2}) + c$

(27) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln (x + \sqrt{x^2 - a^2}) + c$

(28) $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x\sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$

(29) $\int \sqrt{x^2 + a^2}\,dx = \frac{1}{2}x\sqrt{x^2 + a^2} + \frac{1}{2}a^2 \ln(x + \sqrt{x^2 + a^2}) + c$

(30) $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2 \ln(x + \sqrt{x^2 - a^2}) + c$

(31) $\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + c$

(32) $\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + c$

(33) $\int e^{ax} \{af(x) + f'(x)\}\,dx = e^{ax} f(x) + c$

(34) Integration by parts:

$$\int UV\,dx = U\int V\,dx - \int\left\{\frac{du}{dx}\int V\,dx\right\}dx$$

(35) Area Under a curve

$$A = \int_a^b y\,dx$$

(36) Trigonometric substitutions

(a) $\int \sqrt{a^2 - x^2}\,dx$ Substitute $x = a \sin\theta$

(a) $\int \sqrt{x^2 + a^2}\,dx$ Substitute $x = a \tan\theta$

(a) $\int \sqrt{x^2 - a^2}$ Substitute $x = a \sec\theta$

CHAPTER 07

CIRCLE

BASIC DEFINITIONS AND FORMULAS

(1) Circle: A circle is the set of points in a plane whose distance from a given fixed point remain constant fixed point is called centre of the circle denoted by C and the constant distance is called the radius of the circle denoted by r.

(2) Theorem: (i) The point (x_1, y_1) lies out side of the circle

$$x^2 + y^2 = r^2 \text{ if } x_1^2 + y_1^2 - r^2 > 0.$$

(ii) The point (x_1, y_1) lies on the circle

$$x^2 + y^2 = r^2 \text{ if } x_1^2 + y_1^2 - r^2 = 0.$$

(iii) The point (x_1, y_1) lies inside of the circle $x^2 + y^2 = r^2$

if $x_1^2 + y_1^2 - r^2 < 0$.

(3) Concentric circles: Circles having the same centre are called concentric circles.

(4) Concyclic: Four points P_1, P_2, P_3 & P_4 are said to be concyclic if the circle passes through these four poionts.

(5) Standard Equation of Circle:

$$(x - a)^2 + (y - b^2) = r^2$$

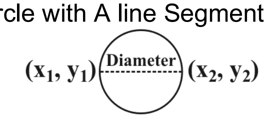
(6) General Equation of Circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $(-g, -f)$

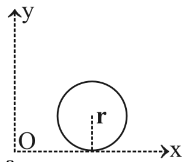
Radius = r = $\sqrt{g^2 + f^2 + c}$

(7) Equation of a Circle with A line Segment as its Diameter:



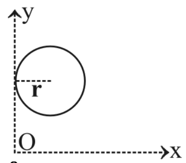
$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(8) Equation of Circle Touching the x–axis with Centre $(-g, -f)$ and Radius = r = $-f$



$$x^2 + y^2 + 2gx + 2fy + g^2 = 0$$

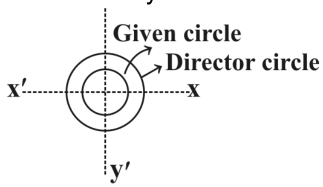
(9) Equation of Circle Touching the y–axis with Center (–g, –f) and Radius = r = –g



$$x^2 + y^2 + 2gx + 2fy + f^2 = 0$$

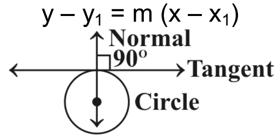
(10) Equation of Director Circle to the Circle $x^2 + y^2 = r^2$ will be

$$x^2 + y^2 + 2r^2$$



(11) Equation of the Tangent:

$$y - y_1 = m(x - x_1)$$

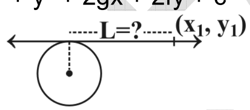


(12) Equation of the Normal:

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

(13) Length of the Tangent Segment From the Point (x_1, y_1) To the Circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

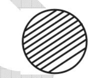


$$L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

(14) Parametric Equation of Circle:

$$x = r\cos\theta \qquad y = r\sin\theta$$

(15) Area of Circle = A



$$A = \pi r^2$$

CHAPTER 08

PARABOLA, ELLIPSE AND HYPERBOLA

BASIC DEFINITIONS AND FORMULAS

PARABOLA: A parabola is the set of all points P in the plane which are equidistance from a fixed line (directrix) and a fixed point (focus) in the plane.

The straight line through the focus and perpendicular to the directrix is called the axis of the parabola.

The point where the parabola meets its axis is called the vertex of the parabola.

ELLIPSE: The sum of the focal distances of a point on an ellipse is constant and equal to the length of major axis.

HYPERBOLA: The locus of a point, the difference of whose distances from two fixed points is constant is hyperbola.

STANDARD PARABOLA

Case I CUP-RIGHT PARABOLA

(1) Vertex, V (0, 0)
(2) Focus, S (a, 0)
(3) End-Points of Latus Rectum
L_1 (a, 2a) and L_2 (a, –2a)
(4) Equation of Directrix $\boxed{x + a = 0}$
(5) Equation of parabola $\boxed{y^2 = 4ax}$

Case II CUP-LEFT PARABOLA

(6) Vertex, V (0, 0)
(7) Focus, S (–a, 0)
(8) End-Points of Latus Rectum
L_1 (–a, 2a) and L_2 (–a, –2a)
(9) Equation of Directrix $\boxed{x - a = 0}$
(10) Equation of parabola $\boxed{y^2 = -4ax}$

Case III CUP-UP PARABOLA

(11) Vertex, V (0, 0)
(12) Focus, S (0, a)
(13) End-Points of Latus Rectum
L_1 (2a, a) and L_2 (–2a, a)
(14) Equation of Directrix $\boxed{y + a = 0}$
(15) Equation of parabola $\boxed{x^2 = 4ay}$
(16) Parametric Equation of parabola

$\boxed{x = at^2}$, $\boxed{y = 2at}$

(17) Length of Latus Rectum = 4a

Case IV CUP-DOWN PARABOLA

(18) Vertex, V (0, 0)
(19) Focus, S (0, –a)
(20) End-Points of Latus Rectum
L_1 (2a, –a) and L_2 (–2a, –a)
(21) Equation of Directrix $\boxed{y - a = 0}$
(22) Equation of parabola $\boxed{x^2 = -4ay}$

GENERAL PARABOLA

Case I PRINCIPAL AXIS OF PARABOLA PARALLEL TO X–AXIS

(23) Vertex, V (h, k)
(24) Focus, S (h + a, k)
(25) End-Points of Latus Rectum
L_1 (h + a, k + 2a) and L_2 (h + a, k – 2a)
(26) Equation of Directrix $\boxed{(x - h) + a = 0}$
(27) Equation of parabola $\boxed{(y - k)^2 = 4a (x - h)}$

Case II PRINCIPAL AXIS OF PARABOLA PARALLEL TO Y–AXIS

(28) Vertex, V (h, k)
(29) Focus, S (h, k + a)
(30) End-Points of Latus Rectum
L_1 (h + 2a, k + a) and L_2 (h – 2a, k + a)
(31) Equation of Directrix $\boxed{(y - k) + a = 0}$
(32) Equation of parabola $\boxed{(x - h)^2 = 4a (y - k)}$

STANDARD ELLIPSE

Case I MAJOR AXIS ALONG X – AXIS

Centre, C (0, 0)
Vertices, A (a, 0), A′ (–a, 0)
Foci, S (c, 0), S′ (–c, 0)
End-Point of Minor Axis
B (0, b), B′ (0, –b)
Equations of Directrices

$\boxed{x = \frac{a}{e}}$, $\boxed{x = -\frac{a}{e}}$

Equation of Ellipse $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

Case II MAJOR AXIS ALONG Y – AXIS

Centre, C (0, 0)
Vertices, A (0, a), A′ (0, –a)
Foci, S (0, c), S′ (0, –c)
End-Point of Minor Axis

B (b, 0), B′ (–b, 0)

Equations of Directrices

$y = \frac{a}{e}$, $y = -\frac{a}{e}$

Equation of Ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

COMMON POINTS FOR ELLIPSE

Length of Major Axis = 2a

Length of Minor Axis = 2b

Length of Latus Rectum = $\frac{2b^2}{a}$

Distance between Foci = 2C

Distance Between directrices = $\frac{2a}{e}$

Relation Between a, b and c $c^2 = a^2 - b^2$

GENERAL ELLIPSE

Case I MAJOR AXIS PARALLEL TO X – AXIS

Centre, C (h, k)

Vertices, A (h + a, k), A′ (h – a, k)

Foci, S (h + c, k), S′ (h – c, k)

End-Point of Minor Axis

B (h, k + b), B′ (h, k – b)

Equations of Directrices

$x - h = \frac{a}{e}$, $x - h = -\frac{a}{e}$

Equation of Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Case II MAJOR AXIS PARALLEL TO Y – AXIS

Centre, C (h, k)

Vertices, A (h, k + a), A′ (h, k – a)

Foci, S (h, k + c), S′ (h, k – c)

End-Point of Minor Axis

B (h + b, k), B′ (h – b, k)

Equations of Directrices

$y - k = \frac{a}{e}$, $y - k = -\frac{a}{e}$

Equation of Ellipse $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

STANDARD HYPERBOLA

Case I TRANSVERSE AXIS ALONG X – AXIS

(1) Center, C (0, 0)

(2) Vertices, A (a, 0), A′ (–a, 0)

(3) Foci, (c, 0), S′ (–c, 0)

(4) End-points of Conjugate Axis

B (0, b) B′ (0, –b)

(5) Equations of Directrices

$x = \frac{a}{e}$, $x = -\frac{a}{e}$

(6) Equation of Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Case II TRANSVERSE AXIS ALONG Y – AXIS

(7) Center, C (0, 0)

(8) Vertices, A (0, a), A′ (0, –a)

(9) Foci, (0, c), S′ (0, –c)

(10) End-points of Conjugate Axis

B (b, 0) B′ (–b, 0)

(11) Equations of Directrices

$y = \frac{a}{e}$, $y = -\frac{a}{e}$

(12) Equation of Hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

(13) Parametric Equations of Hyperbola

$x = a\sec\phi$, $y = b\tan\phi$

GENERAL HYPERBOLA

Case I TRANSVERSE AXIS PARALLEL TO X – AXIS

(14) Centre, C (h, k)
(15) Vertices, A (h + a, k), A′ (h – a, k)
(16) Foci, S (h + c, k), S′ (h – c, k)
(17) End-Points of Conjugate Axis,
B (h, k + b), B′ (h, k – b)
(18) Equations of Directrices

$x - h = \frac{a}{e}$, $x - h = -\frac{a}{e}$

(19) Equation of Hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Case II TRANSVERSE AXIS PARALLEL TO Y – AXIS

(20) Centre, C (h, k)
(21) Vertices, A (h, k + a), A′ (h, k – a)
(22) Foci, S (h, k + c), S′ (h, k – c)
(23) End-Points of Conjugate Axis,
B (h + b, k), B′ (h – b, k)
(24) Equations of Directrices

$y - k = \frac{a}{e}$, $y - k = -\frac{a}{e}$

(25) Equation of Hyperbola $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

COMMON POINTS FOR HYPERBOLA

(26) Length of Transverse Axis = 2a
(27) Length of Conjugate Axis = 2b
(28) Length of Latus Rectum = $\frac{2b^2}{a}$

(29) Eccentricity = $e = \frac{c}{a}$
(30) Distance between Foci = 2c
(31) Distance Between directrices = $\frac{2a}{e}$
(32) Relation Between a, b and c $c^2 = a^2 + b^2$

CHAPTER 09

VECTORS

BASIC DEFINITIONS AND FORMULAS

(1) Vector: A vector is a physical quantity which has magnitude as well as direction

(2) Position vector: $\overrightarrow{OP} = (x_2 - x_1)\,i + (y_2 - y_1)\,j + (z_2 - z_1)\,k$

(3) Magnitude: $|\overrightarrow{OP}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(4) Direction Cosines: $\cos\alpha = \frac{x}{r}$, $\cos\beta = \frac{y}{r}$, $\cos\gamma = \frac{z}{r}$

and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(5) Triangle law: If two vectors are represented by the sides $\vec{A}$ and $\vec{B}$ of a triangle, taken in the same order, then their resultant is represented by the third side $\vec{C}$. taken in the reverse order.

(6) Law of Parallelogram:

(i) $|a + b| = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$

(ii) $|a - b| = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$

(iii) $\tan\theta = \dfrac{b\sin\alpha}{a + b\cos\alpha}$

(7) Ratio Theorem: $\boxed{r = \dfrac{ma + lb}{l + m}}$

(8) The Scalar (or Dot) Produt of Two vectors

(i) $\vec{a} \cdot \vec{b} = ab\cos\theta$ or $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

(ii) $\vec{a} \cdot \vec{b} = (a_1i + a_2j + a_3k).(b_1i + b_2j + b_3k)$

$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

(9) The vector (or Cross) product of Two vectors

(i) $\vec{a} \times \vec{b} = ab\sin\theta\,\hat{n}$ where $\hat{n}$ is perpendicular $\vec{a}$ and $\vec{b}$

(ii) $\vec{a} \times \vec{b} = (a_1i + a_2j + a_3k) \times (b_1i + b_2j + b_3k)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

(10) Unit vector in the direction of $\vec{a}$ **:** $\boxed{\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}}$

(11) Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ **:** $\hat{c} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

(12) Area of a Parallelogram:

Area = $|\vec{a} \times \vec{b}|$

(13) Area of a triangle:

Area = $\frac{1}{2}|\vec{a} \times \vec{b}|$

(14) Sin (a, b) = $\dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

(15) Scalar product of Three vectors

$\vec{a} \cdot \vec{b} \times \vec{c} = [\vec{a}, \vec{b}\ \vec{c}]$

$\vec{a} \cdot \vec{b} \times \vec{c} = [a_1i + a_2j + a_3k, b_1i + b_2j + b_3k, c_1i + c_2j + c_3k,]$

$$\vec{a} \cdot \vec{b} \times \vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(16) Volume of a parallelopiped:

$V = \vec{a} \cdot \vec{b} \times \vec{c} = [\vec{a}, \vec{b}, \vec{c}]$

(17) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are Coplanar or collinear

then $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ or $[\vec{a}, \vec{b} \times \vec{c}] = 0$

(18) Work = Force. Displacement

OR Work = $\vec{F} \cdot \vec{r}$

(19): $[x_1a + x_2b + x_3c, y_1a + y_2b + y_3c, z_1a + z_2b + z_3c]$

$$= \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} [a, b, c]$$