XII MATHEMATICS

CHAPTER # 01 **FUNCTIONS AND LIMITS** **(Assignment #01)**

Not everything that counts can be counted. Not everything that can be counted counts. —Albert Einstein

Multiple Choice Questions:

1. $\lim_{x\to 0}\frac{\sin\frac{4}{5}x}{x} =$.
   a. $\frac{5}{4}$ b. $\frac{4}{5}$ c. $\frac{1}{4}$ d. $\frac{1}{5}$
2. A function $(x) = \frac{x}{|x|}, x \neq 0$ is __________.
   a. Even function b. odd function c. circular function d. neither even nor odd function
3. $\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n =$ ____________.
   a. 0 b. ∞ c. e d. 1
4. $\lim_{x\to a}\frac{x^n-a^n}{x-a} =$ ___________.
   a. 1 b. na^{n-1} c. n d. 0
5. $f(x) = \sin x + \cos x$ is a/an __________.
   a. Even function b. odd function c. modulus function d. neither even nor odd
6. $\lim_{x\to 0}\frac{\sin\frac{7}{3}x}{x} =$_____________.
   a. $\frac{7}{3}$ b. 7 c. $\frac{3}{7}$ d. 3
7. $\lim_{x\to 2}\frac{x^2-4}{x-2} =$ ___________.
   a. 8 b. 4 c. not defined d. 0
8. $\lim_{x\to 0}\frac{e^x-1}{x} =$ ___________.
   a. 0 b. 1 c. e d. none of these
9. $\lim_{x\to 0}\frac{a^x-1}{x} =$ ___________.
   a. 0 b. 1 c. a d. lna

Short Question:

Q.1 Let If $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by: $f(x) = \begin{cases} 0, & \text{when } x \in \mathbb{Q} \\ 1, & \text{when } x \in \mathbb{R}-\mathbb{Q} \end{cases}$, ($\mathbb{Q}$ being the set of rationals)

Find (i) $f(\sqrt{3})$, $f\left(\frac{1}{5}\right)$, $f\left(\frac{22}{7}\right)$, $f(2)$, $f(\pi)$, $f(1.5)$

Q.2 Define Even and Odd functions. Find whether the given function is even, odd or neither.

(i) $f(x) = \sin x - \tan x$ (ii) $f(x) = \frac{e^x-1}{e^x+1}$ (iii) $f(x) = \begin{cases} \frac{x}{|x|}, & \text{when } 0 \neq x \in \mathbb{R} \\ 0, & \text{when } x = 0 \end{cases}$

Q.3 Find the limit of the sequences:

(i) $\frac{1.2}{3.4}, \frac{3.4}{5.6}, \frac{5.6}{7.8}, \ldots$ (ii) $\frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \ldots$

Q.4 Find whether the sequence $\{a_n\}$ is divergent or convergent.

(i) $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots$ (ii) $1-\frac{3}{4}+\frac{9}{16}-\frac{27}{64}+\ldots$

Q.5 Evaluate:

(i) $\lim_{x\to 0}\frac{\sin^2\left(\frac{x}{3}\right)}{x^2}$ (ii) $\lim_{x\to 0}\frac{\operatorname{cosec}x-\cot x}{x}$ (iii) $\lim_{x\to 0}\frac{1-\cos 5x}{1-\cos 7x}$ (iv) $\lim_{x\to 0}\frac{1-\cos x}{\sin x}$ (v) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

(vi) $\lim_{x\to 0}\frac{\sin 5x-\sin 3x}{x}$ (vii) $\lim_{x\to 0}\frac{\tan x-\sin x}{x}$ (viii) $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$ (ix) $\lim_{x\to 0}\frac{1-\cos px}{1-\cos qx}$ (x) $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{\cos^3 x}$

Q.6 Evaluate:

(i) $\lim_{x\to\infty}\frac{x^2-5x+2}{5x^2+6x-4}$ (ii) $\lim_{x\to 3}\frac{9-x^2}{4-\sqrt{x^2+7}}$ (iii) $\lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{a}$ (iv) $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ (v) $\lim_{x\to\infty}\frac{\sqrt{2x^2+x}}{3x+5}$

(vi) $\lim_{x\to 0}\frac{e^{mx}-e^{nx}}{x}$ (vii) $\lim_{x\to 0}\frac{3e^x-e^{-x}-2}{x}$ (viii) $\lim_{x\to 0}\frac{9e^x-e^{-x}-8}{x}$ (ix) $\lim_{x\to\infty}\frac{\ln(1+e^{-x})}{x}$ (x) $\lim_{x\to-\infty}\frac{2^x-1}{x}$

Q.7 Evaluate:

(i) $\lim_{x\to 1}\left[\frac{1}{1-x}-\frac{3}{1-x^3}\right]$ (ii) $\lim_{x\to 1}\left[\frac{2}{1-x^2}-\frac{1}{x-1}\right]$ (iii) $\lim_{x\to\infty}\left[\frac{x}{1+x}\right]^x$ (iv) $\lim_{x\to\infty}x\left(\sqrt{x^2+1}-x\right)$

(v) $\lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - sinx}{\Delta x}$ (vi) $\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - cosx}{\Delta x}$ (vii) $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$ (viii) $\lim_{x \to 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$

(ix) $\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x-1}$ (x) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$

CHAPTER # 02 THE STRAIGHT LINE (Assignment #02)

A Curve does not exist in its full power until contrasted with a straight line. —Robert Henry

Multiple Choice Questions:

1. If a line is perpendicular to y-axis then its equation is _________.
   a. $x = 0$ b. y = constant c. x =constant d. $y = 0$
2. The point of intersection of internal bisectors of angles of triangle is called _________.
   a. In-centre b. centroid c. ortho-centre d. circum-centre
3. Distance of the point (4,5) from the $y -$ axis is ___________.
   a. 5 units b. 4 units c. 9 units d. 1 unit
4. If a line is parallel to x −axis, its equation is __________.
   a. $x = 0$ b. $y = 0$ c. x =constant d. y =constant
5. If a straight line is parallel to y −axis then its slope is _________.
   a. 1 b. 0 c. -1 d. ∞
6. If a straight line is parallel to x −axis then its slope is _________.
   a. 1 b. 0 c. -1 d. 2
7. The centroid of the triangle whose vertices are A(0,0), B(1,0) and C(0,1) is __________.
   a. (1,1) b. (0,0) c. $\left(\frac{1}{3},\frac{1}{3}\right)$ d. $\left(\frac{1}{2+\sqrt{2}},\frac{1}{2+\sqrt{2}}\right)$
8. The in-centre of the triangle whose vertices are A(0,0), B(1,0) and C(0,1) is __________.
   a. (1,1) b. (0,0) c. $\left(\frac{1}{3},\frac{1}{3}\right)$ d. $\left(\frac{1}{2+\sqrt{2}},\frac{1}{2+\sqrt{2}}\right)$
9. The x and y intercepts of $3x - 5y = 15$ are ____________.
   a. 5 and −3 b. −5 and −3 c. 5 and 3 d. none of these
10. The slope of the line joining the pair of points (−1, 2) and (−1, 5) is __________.
    a. 0 b. ∞ c. 3 d. none of these

Short-Answer Questions:

Q.1 Find the point which is equidistant from (0,0), (3,1) and (6,0).

Q.2 The centroid of a triangle whose two vertices are (2,4) and (3, -4) is found to be (3,1). Find the third vertex.

Q.3 Find the ratio in which y −axis divides the join of (-5,3) and (8,6). Also find the coordinates of point if division.

Q.4 In what ratio does the point M(2,4) divides the join of L(7,9) and N(-1,1).

Q.5 The line join the points P(1,-2) and Q(-3,4) is trisected. Find the point of trisection.

Q.6 A straight line passes through the points A(-12, -13) and B(-2, -5). On this line, find a point whose ordinate is equal to -1.

Q.7 Prove that the diagonals of an isosceles trapezoid are equal.

Q.8 Find the equation of the locus of a moving point such that the slope of the line joining the point to A(1,3) is three times the slope of the line joining the point to B(3,1).

Q.9 The line through (6,-4) and (-3,2) is parallel to the line through (2,1) and (0,y). Find y also the equation of both the lines.

Q.10 The line through (2,5) and (-3,-2) is perpendicular to the line through (4,-1) and (x,3). Find x.

Q.11 Show that the equilateral triangle has congruent angles.

Q.12 Find the equation of the perpendicular bisectors of segment joining (15,14) and (-3,-4).

Q.13 Prove that the points whose coordinates are respectively (5,1), (1,-1) and (11,4) on a straight line on the axis.

Q.14 The angle from the line through (2,7) and (-6,5) to a line through (1,-4) is 135^0. Find the equation of second line.

Q.15 The x −intercept of a line is the reciprocal of its y −intecept. The line passes through (2,-1). Find its equation.

Q.16 Find the equation of the straight line which passes through the point (3,-4) and is such that the portion of it between the axes is divided by the point in the ratio 2:3.

Q.17 Find the equation of the line which passes through the point (3,4) and makes intercepts on the axes such that y −intercept is twice that of the x −intercept.

Q.18 If the points (a, b), (a`, b`) (a-a`, b-b`) are collinear, show that their join passes through the origin and that ab` = a`b.

Q.19 Determine the equation of the line which passes through the (-1,2) and has sum of its intercepts equal to 2.

Detailed-Answer Questions:

Q.20 Find the equation of the straight line passing through the point (a,b) such that the portion of the straight line between the axes is bisected at the point.

Q.21 The vertices A,B,C of a triangle are (2,1), (5,2) and (3,4) respectively. Find the coordinates of the circum-centre and also the radius of the circum-circle of the triangle.

Q.22 An equilateral triangle has one vertex at the point (3,4) and another at the point (-2,3). Find the coordinates of the third vertex.

Q.23 Obtain the coordinates of the centroid of the triangle whose vertices are (-2,5), (4,-1) and (5,4). Also find the length of the medians.

Q.24 Find the coordinates of the in-centre of the triangle whose angular points are (-36,7), (20,7) and (0,-8).

Q.25 The points L(3,2), M(4,5) and N(2,4) are the mid-points of the sides of a triangle. Find its vertices.

Q.26 Find the angles of the triangle whose vertices are A(-2,1), B(4,-3) and C(6,4).

Q.27 Prove that if the diagonals of a parallelogram are perpendicular, the figure is rhombus.

Q.28 Show that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to one-half its length.

Q.29 A is two-third the way from (1,10) to (-8,4) and B is the mid-point of (0,-7) and (6, -11). Find the distance $|\overline{AB}|$.

Q.30 Find the slope of the line through the mid-point of the segment from A(-4,4) to B(2,2) and the point which is three-fifth the way from C(5,3) and D(-3,-2).

Q.31 A is the mid-point of the segment bounded by (-2,3) and (6,-1), B is a point at $\frac{3}{4}$ of the distance from (4,3) and (0,-3). Find the equation of $\overline{AB}$.

CHAPTER # 03 THE GENERAL EQUATION OF STRAIGHT LINE (Assignment # 03)

Every line tells its own story, even the very tentative ones. —Gillian Redwood

Multiple Choice Questions:

1. The line 4x+5y+2=0 is perpendicular to the line:
   a. 5x+4y-2=0 b. 5x-4y+3=0 c. 4x+5y-2=0 d. -5x-4y+2=0
2. Two lines represented by $ax^2+2hxy+by^2=0$, are perpendicular to each other, if:
   a. a+b=0 b. a-b=0 c. a=0 d. b=0
3. Area of triangle ABC, when A,B,C are collinear is _________.
   a. ∞ b. zero c. positive d. negative
4. The angle between the pair of lines $3x^2+8xy-3y^2=0$ is _________.
   a. 90^0 b. 45^0 c. 0^0 d. 180^0
5. The line 2x+3y+6=0 is parallel to the line:
   a. 2x+3y-8=0 b. 2x-3y+7=0 c. x-y+6=0 d. 3x-2y+9=0
6. If the two lines ax-2y=1 and 6x-4y=b are parallel then value of a is __________.
   a. 3 b. 2 c. $-\frac{4}{3}$ d. $\frac{4}{3}$
7. The point which lies below the line 2x-3y+4=0 is __________.
   a. (-8, -3) b. (10, -5) c. (-35, 9) d. none of these
8. The distance between the parallel lines 3x+4y+10=0 and 6x+8y-9=0 is __________.
   a. 2.9 units b. 2.8 units c. 2.7 units d. 3 units
9. The area of triangle ABC whose vertices are A(0,0), B(1,0) and C(0,1) is _________.
   a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. none of these
10. The area of a quadrilateral ABCD whose vertices are A(0,0), B(0,1), C(0,1) and D(1,1) is ________.
    a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. none of these

Short-Answer Questions:

Q.1 Find the equation of the straight line passing through (4,5) and perpendicular to 3x-2y+5=0.

Q.2 Find the equation of the straight line passing through (2, -1) and making acute angle of $\frac{\pi}{4}$ radians with the line 6x+5y=0.

Q.3 Show that the following lines are concurrent also find their point of concurrency.
5x+y+11=0, x+7y+9=0, 2x+y+5=0

Q.4 For what value of k will the three lines 2x-3y-7=0, 4x-3y-11=0, 2x+ky+1=0 be concurrent.

Q.5 Find the coordinates of the foot of the perpendicular from (-2,5) to x+3y+11=0.

Q.6 The point P (-1, 3) is the foot of the perpendicular dropped from the origin to a straight line. Write the equation of this line also find the length of the perpendicular distance.

Q.7 Find the value of 'a' and 'b' for which the line (a + 2b – 3) x + (2a – b + 1) y+6a+9=0 is parallel to the axis of x and has y-intercept = - 3. Also write the equation of this line.

Q.8 Find the value of k for which the two lines (k-1)x+ky-5=0 and kx+(2k-1)y+7=0 intersect at a point lying on the axis of x.

Q.9 Find the distance between the parallel lines: 3x+4y+10=0 and 6x+8y-9=0.

Q.10 Find the value of k when the vertices of the triangle are the points (2,6), (6,3) and (4,k) and area is 14 square units.

Q.11 If Δ denotes the area of a triangle and the coordinates of the points A,B,C,D are (6, 3), (-3, 5), (4, -2) and (x, 3x) and $\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$; find x.

Q.12 The coordinates of two points A and B are (3,4) and (5,-2) respectively. Find the coordinates of any point P if $|\overline{PA}| = |\overline{PB}|$ and the area of triangle PAB is 10 square units.

Q.13 D, E, F are the mid-points of the sides BC, CA and AB respectively of the triangle ABC. Prove that Δ ABC = 4 Δ DEF.

Q.14 Show that the equation of the line through the origin, making an angle of measure ∅ with the line y = mx + b is $\frac{y}{x} = \frac{m+tan\emptyset}{1+mtan\emptyset}$

Q.15 The point (2, -5) is the vertex of a square, one of a square, one of whose sides lies on the line x – 2y – 7 = 0 calculate the area of the square.

Q.16 Given that 3x–2y–5 = 0, 2x+3y+7=0 are the equations of two sides of a rectangle, and that (-2, 1) is one of the vertices; calculate the area of the rectangle.

Q.17 If A(2, 3), B(3, 5) are fixed points and a point P moves such that Δ PAB = 8 sq. units, find the equation of the locus of P.

Detailed-Answer Questions:

Q.18 A line whose y-intercept is 1 less than its x-intercept forms with the coordinate axes a triangle of area 6 square units. What is its equation?

Q.19 Find the equation of the straight line through the intersection of the lines:

(i) l_1: 3x -4y + 1 =0, l_2: 5x + y – 1 = 0 and cutting off equal intercepts from the axes.
(ii) l_1: 43x+29y+43=0, l_2: 23x+8y+6=0 and having a y-intercept = -2 OR
(iii) l_1: 2x+7y-8=0, l_2: 3x+2y+5=0 and making and angle of 45^0 with the line 2x+3y-7=0.

Q.20 The sides of a triangle are given by l_1: 4x-y-7=0, l_2: x+3y-31=0 and l_3: x+5y-7=0. Find the point of intersection of its altitudes (ortho-centre).

Q.21 Find the equation of the straight line through the point of intersection of the lines
3x + 2y + 5 = 0 and 2x + 7y – 8 = 0, bisecting the join of (-1, -4) and (5, -6).

Q.22 Find the in-centre of the triangle, the equation of whose sides are $x = 3$, $y = 4$ and 4x+3y=12.

Q.23 Find the measures of the angle of the triangle, the equation of whose sides are x+y–5 = 0, x – y + 1 = 0 and y = 1 Also find its area.

Q.24 Find the combined equation of the pair of lines through the origin which are perpendicular to the lines represented by $2x^2 - 5xy + y^2 = 0$ $6x^2 - 13xy + 6y^2 = 0$.

Q.25 The gradient of one of the lines of $ax^2+2hxy+by^2=0$ is:
(i) twice that of the other, show that $8h^2 = 9ab$.
(ii) thrice that of the other, show that $5h^2 = 9ab$.
(iii) five times that of the other, show that $3h^2 = 4ab$.

Q.26 Find the centroid of the triangle, the equations of whose sides are $12y^2 - 20xy + 7x^2 = 0$ and 2x – 3y + 4 = 0.

Calculus required continuity, and continuity was supposed to require the infinitely little; but nobody could discover what the infinitely little might be." — Bertrand Russell

Multiple Choice Questions:

1. If $y = \log_a x$, then $dy =$ __________.
   a. $\frac{1}{x} lna\, dx$ b. $\frac{1}{xlne}\ dx$ c. $\frac{1}{xlna}\ dx$ d. $\frac{1}{x} a\, dx$
2. If $f(x) = tan^{-1}3x$, then $f'(x) =$ ____________.
   a. $\frac{1}{1+9x^2}$ b. $\frac{1}{9+x^2}$ c. $\frac{3}{1+9x^2}$ d. $\frac{3}{1+3x^2}$
3. If $f(x) = \ln(sinx)$, then $\frac{dy}{dx} =$ ___________.
   a. $\frac{1}{sinx}$ b. $cosx$ c. $cotx$ d. $tanx$
4. If $f(x) = \tan^{-1}2x$, then $\frac{dy}{dx} =$ ___________.
   a. $\frac{1}{1+x^2}$ b. $\frac{1}{4+x^2}$ c. $\frac{1}{1+4x^2}$ d. $\frac{2}{1+4x^2}$
5. If $f(x) = \cot x$, then $dy =$ ___________.
   a. $cosecxdx$ b. $cosec^2x$ c. $-cosec^2xdx$ d. cot^2xdx
6. $\lim_{x\to 0} \frac{f(x)-f(a)}{x-a}$ is equal to ___________.
   a. $f'(x)$ b. $f'(a)$ c. $f'(0)$ d. $f'(1)$
7. $\frac{d}{dx}(sin^2x + cos^2x) =$ ___________.
   a. 1 b. $2sinxcosx$ c. $22sinxcosx$ d. 0
8. If $f(x) = \tan 9x$, then $f'(x) =$ ___________.
   a. sec^29x b. $9sec^2x$ c. $9sec^29x$ d. $-sec^29x$
9. If $f(x) = lnx^3$, then $f'(x) at\ x = -2$ is ___________.
   a. $\frac{2}{3}$ b. $-\frac{3}{2}$ c. $-\frac{2}{3}$ d. 1
10. If $f(x) = e^{3x+2}$, then $f'(x) =$ ___________.
    a. e^{3x+2} b. $3e^{3x+2}$ c. $2e^{3x+2}$ d. none of these
11. If $f(x) = a^{2x+3}$, then $f'(x) =$ ___________.
    a. $a^{2x+3}lna$ b. $2a^{2x+3}lna$ c. $3a^{2x+3}lna$ d. none of these
12. If $x = cost, y = sint$ then $\frac{dy}{dx} =$ ___________.
    a. tant b. $-cost$ c. cot(t) d. none of these

Short-Answer Questions:

Q.1 Find the first derivative by the first principle at any point x.

(a) $f(x)$=sinx (b) $f(x)$=cosx (c) $f(x)$=tanx (d) $f(x)$=cotx (e) $f(x)$=secx (f) $f(x)$=cosecx
(g) $f(x)$=$\sin^2$x (h) $f(x)$=$\sin^2$x (i) $f(x)$=$\tan^2$x (j) $f(x)$=$\cot^2$x (k) $f(x)$=$\sec^2$x (l) $f(x)$=cosec^2x
(m) $f(x)$=sin2x (n) $f(x)$=cos2x (o) $f(x)$=sin$\sqrt{x}$ (p) $f(x)$=tann$\sqrt{x}$ (q) $f(x)$=cos$\sqrt{x}$ (r) $f(x)$=sinx^2
(s) $f(x)$=cosx^2 (t) $f(x)$=$2x^2-x$ (u) $f(x)$=$3x^3-x$ (v) $f(x)$=$x^{\frac{2}{3}}$

Q.2 Find $\frac{dy}{dx}$ in the following:

(i) $y = \sqrt{(x^2+2x+3)^3}$ (ii) y = 3^{3x^2+x} (iii) y = $ln\left(\frac{e^x}{1+e^x}\right)$ (iv) y = x^x (v) y = $cot^{-1}\left(\frac{2x}{1-x^2}\right)$
(vi) y = $\sqrt{1+x^2}$ +$cot^{-1}x$ (vii) y = $\frac{5x^2-1}{5x^2} + ln\sqrt{1+x^2}$ +$cot^{-1}x$ (viii) y = $\frac{3x^2-1}{3x^2} + ln\sqrt{1+x^2}$ +$tan^{-1}x$
(ix) y = $tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (x) y = x^{cosecx} (xi) y = $sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ (xii) y = $(lnx)^{tan^{-1}x}$ (xiii) y = $x^x + (lnx)^{sinx}$
(xiv) y = $(lnx)^{sinx}$ (xv) y = $(tan^{-1}x)^{cosx}$ (xvi) y = $x^x - x^{cosx}$ (xvii) y = $x^{sinx} + (tanx)^x$
(xviii) y = ln (sec2x+tan2x) (xix) y = $(tanx)^x + x^{tanx}$ (xx) y = $acot^{-1}\{mtan^{-1}(bx)\}$

Q.3 Find $\frac{dy}{dx}$ in the following: (i) y – xy – siny=0 (ii) $\sin(x+y) = \ln(x-y)$ (iii) $\sqrt{x^2+y^2}$ = ln ($x^2 - y^2$)
(iv) $e^x lny = sin^{-1}y$ (v) $2x^2+7y^2+2xy-2x+4y+9=0$ (vi) x$\sqrt{1+y}$ +y$\sqrt{1+x}$ = a (vii) $x^y.y^x = 1$
(viii) $x^y.y^x = a$ (ix) $x^3+y^3+ax^2y+bxy^2=0$ (x) $2x^2-3xy+y^2=0$

Q.4 Find $\frac{dy}{dx}$.

(i) x=cost, y=sint at $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ (ii) x= $acos^23\theta$, y=b$sin^23\theta$ at $\theta=\frac{\pi}{6}$ (iii) x= a(θ - sinθ), y=a(1 – cosθ) at $\theta=\frac{\pi}{2}$
(iv) x= lnt+sint, y= e^t+cost (v) x=$\sin t^3 + \cos t^3$, y=sint+2$\cos^{-1}$t (vi) x= $acos^n\theta$, y=b$sin^n\theta$

(vii) x= $a cos^2 2\theta$, y=$b sin^2 2\theta$ (viii) x= $a cos^3 2\theta$, y=$b sin^3 2\theta$ (ix) x=a(t-tsint), y=a(1-cost) at t=$\frac{\pi}{2}$

(x) x=e^{-t}cost, y=e^{t}sint (xi) x=$tant^3$+$sect^3$, y=tant+2sect

CHAPTER # 05 APPLICATION OF DIFFERENTIAL CALCULUS (Assignment #05)

The only way to learn mathematics is to do mathematics. — Paul Halmos

Multiple Choice Questions:

1. The necessary condition for $f(x)$ to have an extreme value is _________.
   a. $f'(x) = 1$ b. $f'(x) = 0$ c. $f(x) = 0$ d. $f''(x) = 1$
2. $f(x)$= sinx + cosx is a/an __________ fundtion.
   a. even b. odd c. neither even nor odd d. modulus
3. If s= $f(t)$, then $\frac{d^2y}{dx^2}$ is __________.
   a. Distance covered at time t b. speed at time t c. acceleration at time t d. velocity at time t
4. The slope of the tangent to the curve y=6x^2 at (1, -1) is _________.
   a. -12 b. 12 c. 15 d. 6
5. The derivative of any constant with respect to independent variable is ____________.
   a. 1 b. 0 c. ∞ d. -1
6. lne is equal to _________.
   a. e b. $ln1$ c. 1 d. 0
7. The slope of the tangent at the point (1,0) to the curve y^2=x is __________.
   a. 1 b. 0 c. ∞ d. does not exist

Short-Answer Questions:

Q.1 Find an approximate value of: $\cos46^0$, $\cos44^0$, $\cos47^0$, $\sin46^0$, $\sin44^0$, $\tan46^0$, $\tan47^0$.

Q.2 Find an approximate value of: $\log_{10}(4.04)$, $\log_{10}(10.1)$.

Q.3 Show that approximate value of $\sqrt{x+\Delta x} = \sqrt{x} + \frac{1}{2\sqrt{x}}\Delta x$

Q.4 Find an approximate value of $\sqrt{4.1}$, $\sqrt{3.9}$, $\sqrt{9.1}$.

Q.5 Find the maximum and minimum values of:

(i) $f(x) = \frac{1}{3}x^3-2x^2+3x+1$ (ii) $f(x)$ =x(x-1)(x-2) (iii) $f(x) = x^3-3x^2+2x+1$ (iv) $f(x) = 2x^3-15x^2+36x+10$

(v) $f(x) = x^3-9x^2+15x+3$ (vi) $f(x) = -x^4+2x^2$ (vii) $f(x) = \frac{(x-2)(x-3)}{x^2}$ (viii) $f(x) = (x-3)^3(x-2)^2$

(ix) $f(x) = e^x sinx$ (x) $f(x) = \sin(2x) - x$

Q.6 Show that the maximum value of $f(x) = \frac{lnx}{x}$ is $\frac{1}{e}$.

Q.7 Show that the maximum value of $f(x) = \frac{x}{lnx}$ is e.

Q.8 Find a right angled triangle of maximum area with a hypotenuse of length 'h'.

CHAPTER # 06 **ANTI-DERIVATIVES** **(Assignment #06)**

Common integration is only the memory of differentiation—Augustus De Morgan

Multiple Choice Questions:

1. $\int e^{sinx} cosx\, dx =$ ___________.
   a. $e^{sinx} + C$ b. $e^{cosx} sinx + C$ c. $e^{sinx} sinx + C$ d. $e^{sinx} + C$
2. $\int \frac{f'(x)}{f(x)} dx =$ ____________.
   a. $\frac{\{f(x)\}^{n+1}}{n+1} + C$ b. $lnf(x) + C$ c. $\frac{1}{f(x)} + C$ d. $lnf'(x)$+C
3. If n= - 1 then $\int \{f(x)\}^n f'(x) dx =$___________.
   a. $\frac{\{f(x)\}^{n+1}}{n+1} + C$ b. $\frac{\{f(x)\}^{n+1}}{n} + C$ c. $\ln f(x) + C$ d. $\frac{\{f(x)\}^{n-1}}{n-1} + C$
4. $\int e^{tanx} sec^2 x\, dx =$ ___________.
   a. $sec^2 x + C$ b. $e^{secx} + C$ c. $e^{tanx} + C$ d. $tanx + C$
5. $\int sin30^0 dx =$ __________.
   a. $cos30^0 + C$ b. $\frac{-cos30^0}{30^0} + C$ c. 0 d. $0.5x + c$
6. $\int x^p dx$, p$\neq -1$ is equal to ___________.
   a. $\frac{x^{p+1}}{p+1} + C$ b. $\frac{x^{p-1}}{p-1} + C$ c. $\frac{x^{p+1}}{p-1} + C$ d. $\frac{x^{p-1}}{p+1} + C$
7. $\int \{f(x)\}^n f'(x) dx =$___________.
   a. $\frac{\{f(x)\}^{n+1}}{n+1} + C$ b. $\frac{\{f(x)\}^{n+1}}{n} + C$ c. $\ln f(x) + C$ d. $\frac{\{f(x)\}^{n-1}}{n-1} + C$
8. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$ ___________.
   a. $e^{\sqrt{x}} + C$ b. $2e^{\sqrt{x}} + C$ c. $\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$ d. none of these
9. An equation involving derivative is called ____________ equation.
   a. Ploynomial b. differential c. exponential d. logarithmic
10. $\int a^x dx$ is equal to ___________.
    a. $\frac{a^x}{lna}$ b. $a^x lna$ c. $\frac{lna}{a^x}$ d. none of these
11. $\int cosecx\, dx =$ ___________
    a. $lntan\left(\frac{x}{2}\right) + C$ b. $lntan\left(\frac{x}{2} + \frac{\pi}{4}\right) + C$ c. $lnsinx + C$ d. $lnsecx + C$

Short-Answer Questions:

Q.1 Evaluate the following:

(i) $\int_0^2 \frac{dx}{\sqrt{1+x} + \sqrt{x}}$ (ii) $\int_{-1}^1 (2x^2 + 4)^3 4x dx$ (iii) $\int_0^2 (x^2 + 3x + 5)^{-\frac{2}{3}} \left(x + \frac{3}{2}\right) dx$ (iv) $\int x^2 \sqrt{4 + x}\, dx$

(v) $\int \frac{x\, dx}{1+\sqrt{x}}$ (vi) $\int (x^3 + 1)^{\frac{7}{5}} x^5 dx$ (vii) $\int e^x sine^x dx$ (viii) $\int e^x cose^x dx$ (ix) $\int \frac{cosecxcotx}{a+bcosecx} dx$

(x) $\int \frac{secxtanx}{a+bsecx} dx$ (xi) $\int x^3 \sqrt{7 + x^2}\, dx$ (xii) $\int_0^{\frac{\pi}{2}} sin^3 x\, dx$ (xiii) $\int sin^4 x\, dx$

(xiv) $\int_0^{\frac{\pi}{6}} sin^5 3x cos^3 3x\, dx$ (xv) $\int sin3xcos2x\, dx$ (xvi) $\int cos5xsin3x\, dx$ (xvii) $\int sin4xsin2x\, dx$

(xviii) $\int_0^{\frac{\pi}{2}} tan^4 x\, dx$ (xix) $\int sec^4 xtan^4 x\, dx$ (xx) $\int tan^2 xsecx\, dx$ (xxi) $\int sec^3 xtan^3 x\, dx$

(xxii) $\int cot^5 xcosec^3 x\, dx$ (xxiii) $\int cot^2 xcosec^4 x\, dx$ (xxiv) $\int cot^2 xcosecx\, dx$ (xxv) $\int_0^{\frac{\pi}{3}} \frac{1}{1-sinx} dx$

(xxvi) $\int sin^5 x\, dx$ (xxvii) $\int_0^{\frac{\pi}{2}} cos2xcosx\, dx$ (xxviii) $\int_{\frac{\pi}{2}}^{\pi} cot^4 \frac{x}{2}\, dx$ (xxix) $\int_1^2 \sqrt[3]{x^3 + x^2 + 7}\,(3x^2 + 2x) dx$

(xxx) $\int_0^{\frac{\pi}{2}} cos^4 x\, dx$ (xxxi) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cos^3 x}{\sqrt{sinx}} dx$ (xxxii) $\int_0^{\pi} tan^3 xsecx dx$ (xxxiii) $\int \frac{e^x dx}{1+e^{2x}}$ (xxxiv) $\int \frac{tanx}{\ln(cosx)} dx$

(xxxv) $\int \frac{cos^2(lnx)}{x} dx$ (xxxvi) $\int \frac{cotx}{\ln(sinx)} dx$ (xxxvii) $\int \frac{secxcosecx}{\ln(tanx)} dx$ (xxxviii) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ (xxxix) $\int_0^{2\sqrt{3}} \frac{x\, dx}{\sqrt{x^2+4}}$

(xxxx) $\int \frac{(2x-3)dx}{x^2+2x+2}$

Detailed-Answer Questions:

Q.2 Evaluate the following:

(1) $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^5\,dx}{\sqrt{9-x^2}}$ (2) $\int_0^2 \frac{x^2dx}{\sqrt{x^2+4}}$ (3) $\int_0^2 \frac{x^3\,dx}{\sqrt{16-x^2}}$ (4) $\int_0^1 \frac{x^3\,dx}{\sqrt{4-x^2}}$ (5) $\int_0^a \frac{dx}{(a+x^2)^{\frac{3}{2}}}$ (6) $\int_0^1 \frac{x^2\,dx}{(4-x^2)^{\frac{3}{2}}}$

(7) $\int \frac{x^3dx}{\sqrt{a^2-x^2}}$ (8) $\int \frac{\sqrt{x^2-a^2}dx}{x}$ (9) $\int x^3\sqrt{x^2-9}\,dx$ (10) $\int_2^3 \frac{dx}{x^2-2x+5}$ (11) $\int_0^1 (x+1)^3\sqrt{x^2+2x+5}dx$

(12) $\int \frac{dx}{x^2+4x+5}$ (13) $\int \frac{dx}{4x-x^2}$ (14) $\int \frac{dx}{\sqrt{4x-x^2}}$ (15) $\int \frac{(x+3)dx}{x^2+2x+5}$ (16) $\int_0^{\frac{\pi}{2}} x\cos x\,dx$ (17) $\int 6x^5e^{x^3}dx$

(18) $\int 2x^3e^{x^2}dx$ (19) $\int e^{ax}\sin bx dx$ (20) $\int e^{ax}\cos bx dx$ (21) $\int e^x\sin 2x dx$ (22) $\int e^{2x}\sin 3x dx$

(23) $\int e^x\cos x dx$ (24) $\int x\ln x dx$ (25) $\int x^2\ln x dx$ (26) $\int (\ln x)^2dx$ (27) $\int_0^{\frac{\pi}{2}} \tan^{-1}x\,dx$ (28) $\int_0^{\frac{\pi}{2}} x\tan^{-1}x\,dx$

(29) $\int x^2\tan^{-1}x\,dx$ (30) $\int x^3\tan^{-1}x\,dx$ (31) $\int \sin^{-1}x\,dx$ (32) $\int x^2\cos^{-1}x\,dx$

(33) $\int x^2\sin^{-1}x\,dx$ (34) $\int e^x\frac{1+\sin x}{1+\cos x}\,dx$ (35) $\int \frac{7x-25}{(x-3)(x-4)}\,dx$ (36) $\int \frac{\cos x}{\sin x(2+\sin x)}\,dx$

(37) $\int \frac{\sin x}{(1+\cos x)(2+\cos x)}\,dx$ (38) $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$ (39) $\int \frac{5\sin x}{6+\cos x-\cos^2x}\,dx$

(40) $\int \frac{\sin x}{2+3\cos x+\cos^2x}\,dx$ (41) $\int \frac{2xdx}{(1+x^2)(3+x^2)}$ (42) $\int \frac{(3x^2+x-2)dx}{(x-1)(x^2+1)}$ (43) $\int \frac{x^2-2}{(x+1)(x-1)^2}\,dx$

(44) $\int \frac{x^2}{(x-1)^3}\,dx$

Q.3 Find the area above the x-axis, under the following curves.

(1) y=tanx, $a=\frac{\pi}{4}$ and $b=\frac{\pi}{3}$ (2) $y=3x^4-2x^3+1$, x=1 and x=2 (3) $x^2+y^2 = 9$, a=-2 and b=1

(4) $\frac{x^2}{4}+\frac{y^2}{9}=1$, a=−1 and b= 1 (5) $y=\tan^3x$, a=$\frac{\pi}{6}$ and b=$\frac{\pi}{4}$ (6) $x^2+y^2 = 4$, a=$\frac{1}{2}$ and b=$\frac{3}{2}$

Q.4 Solve the differential equation:

Solving the differential equation means finding the actual function from which this derivative has been obtained. The solution in which value of 'C' is not know is 'General Solution' while the other is 'Particular Solution.'

(1) $\frac{dy}{dx}$ = x + sinx, y = 3 when x = 0 (2) $\frac{dy}{dx} = \sin^2y\cos^2x\sin x$ (3) $\frac{dy}{dx} = \frac{\sin^2y}{\cos^2x}$

(4) $y(1+x^2)\frac{dy}{dx} = x(1+y^2)y^2$ (5) $\frac{dy}{dx} = \frac{\sqrt{1-\cos y}}{\sin y}, y(3)=\frac{\pi}{2}$ (6) $\frac{dy}{dx} = \sqrt{y+1}\,\sqrt{3x+1}$, y=3 when x=5

(7) $y\frac{dy}{dx} = x(y^4+2y^2+1)$ and y(-3) = 1 (8) $\frac{dy}{dx} = \sqrt{xy}$, y = 100 when x = 9

(9) $\frac{dy}{dx} = \sqrt{xy-2y-3x+6}$, y=12 when x=6 (10) $2+2y\frac{dy}{dx} = 1+3x^2$, y(2) = 1

CHAPTER # 07 CIRCLE (Assignment #07)

Geometry is the science of correct reasoning on incorrect figures." — George Polya

Multiple Choice Questions:

1. The centre of circle $2x^2+2y^2+8x=0$ is ________.
   a. (0,0) b. (-4,0) c. (8,0) d. (-2,0)
2. The length of tangent from the point (2,4) to the circle $x^2+y^2-5=0$ is ________ unit.
   a. 5 b. 15 c. $\sqrt{15}$ d. $\sqrt{5}$
3. Which of the following circles passes through the origin?
   a. $x^2+y^2+8x+7=0$ b. $x^2+y^2-9y+11=0$ c. $x^2+y^2+8x+11y=0$ d. $x^2+y^2-8x+11y+19=0$
4. The centre of the circle $x^2+y^2+6x-10y+33=0$ is ______.
   a. (-3,5) b. (-3,-5) c. (3,5) d. (3,-5)
5. The centre of the circle $x^2+y^2-6x+8y-24=0$ is __________.
   a. (3,-4) b. (-3,4) c. (4,3) d. (3,4)
6. The length of the tangent from the point (-2,3) to the circle $x^2+y^2+3=0$ is __________.
   a. 3 b. 4 c. 5 d. 6
7. The radius of circle $x^2+y^2-2x+6y-15=0$ is ________.
   a. 5 b. 4 c. 3 d. 6
8. The equation of the circle through the points (0,0), (1,0) and (0,1) is __________.
   a. $x^2+y^2+x+y=0$ b. $x^2+y^2-x-y=0$ c. $x^2+y^2+x+y+1=0$ d. $x^2+y^2-x-y-1=0$
9. The equation of the circle touching each axis at a distance 5 units from the origin is ________.
   a. $(x-5)^2+(y-5)^2=(5)^2$ b. $(x+5)^2+(y-5)^2=(5)^2$ c. $(x+5)^2+(y+5)^2=(5)^2$ d. All of these
10. Which point lies on the circle $x^2+y^2=49$.
    a. (3,-2) b. (4,8) c. $(-5, -2\sqrt{6})$ d. none of these

Short-Answer Questions:

Q.1 Prove that the equation of the circle through the points (p,0), (q,0), (0,r) is $r(x^2+y^2)-r(p+q)x-(r^2+pq)+pqr=0$.

Q.2 Show that the four points (3,4), (-1,-4), (-1,2) and (3,-6) are concyclic and find the equation of the circle on which they lie.

Q.3 Find the equation of the circle which passes through the point (-2,-4) and has the same centre as the circle whose equation is $x^2+y^2-4x-6y-23=0$.

Q.4 Find the equation of the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the axes.

Q.5 Find the equation of the circle of radius a units which passes through the two points on the axis of x which are at a distance b unit from the origin.

Q.6 Find the equation of the circle touching each axis at a distance 6 unit from the origin in 4th quadrant.

Q.7 Find the equation of the circle which touches x −axis and passes through the points (1,-2) and (3,-4).

Q.8 Find the equation of the circle which passes through the two points (a,0) and (-a,0) and whose radius is $\sqrt{a^2+b^2}$.

Q.9 Find the equation of the circle containing the point (6,0) and touching the line $x = y$ at the point (4,4).

Q.10 Find the equation of the circle which passes through the points (-1,-1) and (3,1) and with centre on the line $x - y + 10 = 0$.

Q.11 Find the equation of the circle containing the points (-1,-2) and (6,-1) and touching the line $y = 0$.

Q.12 Find the equation of the circle concentric with the circle $x^2+y^2-4x-6y-23=0$ and touching y −axis.

Q.13 Prove that the curves $x^2+3y^2-24=0$ and $3x^2+y^2=12$ intersect at right angle at the point $(\sqrt{6}, \sqrt{6})$.

Q.14 Find the condition that the conic $ax^2 + by^2 = 1$ should cut $a'x^2 + b'y^2 = 1$ orthogonally.

Q.15 Obtain the condition of tangency of $y = mx + c$ with the circles in the standard form.

Q.16 Find the equation of tangent to $x^2+y^2-6x-2y+9=0$ at the point (1,2).

Q.17 Prove the product of abscissa of the points where $x^2+y^2+2gx+2fy+c=0$ is equal to $\frac{c}{1+m^2}$.

Q.18 Find the equation of the circle concentric with the circle $x^2+y^2-8x+12y-12=0$ and passes through the point (5,4).

Q.19 Prove that the two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other if $\frac{1}{f^2} + \frac{1}{g^2} = \frac{1}{c}$

Detailed-Answer Questions:

Prove analytically:

Q.20 The tangent to a circle is perpendicular to the radial segment at the point of contact.

Q.21 A normal to a circle from the centre of the circle.

Q.22 The perpendicular from the centre of the circle to its chords bisects the chord.

Q.23 Congruent chords of a circle are equidistant from the centre.

CHAPTER # 08 PARABOLA, ELLIPSE AND HYPERBOLA (Assignment #08)

Problems cannot be solved at the same level of awareness that created them.—Albert Einstein

Multiple Choice Questions:

1. The length of latus rectum of parabola $x^2 = 4ay$ is _______.
   a. 4a b. a c. 4 d. $|4a|$
2. If $b^2=a^2(e^2-1)$, then conic is _________.
   a. parabola b. ellipse c. hyperbola d. circle
3. The distance between foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **is** ________.
   a. $2a$ b. $2c$ c. $2b$ d. $2\frac{a}{e}$
4. The distance between foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ **is** ________.
   a. $2a$ b. $2c$ c. $2b$ d. $2\frac{a}{e}$
5. The vertex of the parabola $(x-1)^2 = 8(y+2)$ is _________.
   a. (1,-2) b. (0,1) c. (2,0) d. (0,0)
6. If $e = 1$, then conic is ________.
   a. parabola b. ellipse c. hyperbola d. circle
7. The vertices of hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ are _____________.
   a. $(\pm 2, 0)$ b. $(0, \pm 2)$ c. $(0, \pm 4)$ d. $(\pm 4,0)$
8. If $e = \frac{3}{2}$, then conic is ________.
   a. parabola b. ellipse c. hyperbola d. circle
9. If $b^2=a^2(1-e^2)$, then conic is _________.
   a. parabola b. ellipse c. hyperbola d. circle
10. The equation of the auxiliary circle of the ellipse $x^2+2y^2=5$ is __________.
    a. $x^2+y^2=5$ b. $x^2+2y^2=2$ c. $x^2+2y^2=\frac{5}{2}$ d. none of these
11. The focus of the parabola $2x^2 = -5y$ is __________.
    a. $\left(0,-\frac{5}{8}\right)$ b. $\left(0,-\frac{5}{4}\right)$ c. $\left(0,-\frac{5}{2}\right)$ d. none of these

Short-Answer Questions:

Q.1 Find the equation of the parabola satisfying the following conditions.
(i) focus (-5,3), directrix $y = 7$ (ii) focus (2,3), directrix $y = 7$

Q.2 Determine the vertex, focus and the equation of directrix of the following parabolas.
(i) $y^2-6y+8x-23=0$ (ii) $x^2+4x+4y-12=0$ (iii) $y^2+4y+3x-92=0$ (iv) $y^2-x-2y-1=0$ (v) $x^2-6x-2y+5=0$

Q.3 Find the equation of the circle whose diameter is the latus rectum of the parabola $x^2= -36y$.

Q.4 Find the eccentricity, semi-axes, centre, vertices and coordinates of foci of the following ellipses and draw their graph. (i) $25x^2+9y^2=225$ (ii) $4x^2-32x+25y^2-300y+864=0$ (iii) $4x^2-16x+25y^2+200y+316=0$

Q.5 Find the equation of the following ellipses whose centre is at origin and which satisfy the given condition.
(i) $(\pm 6, 0)$ and latus rectum of length 3 (ii) vertices at $(0, \pm 5)$ and passing through the point $\left(\frac{4}{5},3\right)$

Q.6 Find the equation of the circle passing through focus of the parabola $x^2+8y=0$ and foci of the ellipse $16x^2+25y^2=400$.

Q.7 Find the equation of the circle passing through focus of the parabola $x^2-8y=0$ and foci of the hyperbola $9x^2-16y^2=144$.

Q.8 Find the length of, and the equation to the focal radii draw to c point $(4\sqrt{3}, 4)$ of the ellipse $25x^2 + 16y^2 = 1600$.

Q.9 Find the distance between the directrices, vertices and foci of the ellipse $9x^2 +13y^2 = 117$.

Q.10 The length of the major axis of an ellipse is 20 and its foci are the points $(\pm 5,0)$; find the equation of the ellipse.

Q.11 An ellipse is drawn to pass through the points (1, 12) (8, 10) and (1, -4) and to have the line x = 4 as an axis of symmetry; find the coordinates of its foci.

Q.12 Find the equation of the following hyperbolas whose centre is at origin and which satisfy the given condition.
(i) focus (8,0) and directrix $x = 4$ (ii) eccentricity 3, focus (6,0)
(iii) length of latus rectum=$\frac{64}{3}$, transverse axis is along y —axis and eccentrivity $\sqrt{3}$.

Q.13 Find the eccentricity, semi-axes, centre, vertices and coordinates of foci of the following.

(i) $25x^2+9y^2=225$ (ii) $4x^2-32x+25y^2-300y+864=0$ (iii) $4x^2-16x+25y^2+200y+316=0$

Q.14 Find the eccentricity of hyperbola whose latus rectum is four times the transverse axis.

Q.15 Find the distance between the directrices of the hyperbola $16x^2-9y^2=144$ and also find the equation of the directrices.

Q.16 Find the coordinates of the centre and of the foci, the length of the semi-transverse axis and the eccentricity of the following hyperbolas.

(i) $4x^2-y^2-8x-2y-1=0$ (ii) $16y^2-9x^2+36x+64y-116=0$ (iii) $9x^2-y^2-36x-6y+18=0$
(iv) $9x^2-16y^2-36x-32y+164=0$ (v) $9x^2-16y^2-18x-64y-199=0$ (vi) $16x^2-36y^2-48x+180y-225=0$
(vii) $9x^2-16y^2-36x-32y-16=0$

Q.17 Prove that the line $lx + my + n = 0$ and the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ have just one point in common if $a^2 l^2 + b^2 m^2 - n^2 = 0$.

Q.18 Find the equation of the tangents at the ends of the latus rectum of the parabola $y^2=4ax$.

Q.19 Prove that the parabolas $x^2=4ay$ and $y^2=4bx$ intersect at angle $Tan^{-1}\frac{3}{2}\left(\frac{a^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{2}{3}}+b^{\frac{1}{3}}}\right)$.

Q.20 Find the conditions that the line $x\cos\alpha + y\sin\alpha = p$ will touch the:

(i) ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (ii) parabola $y^2 = 4ax$

Q.21 Show that the tangents at the ends of the latera recta of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ have slopes $\pm e$.

Q.22 If (x_1, y_2) , (x_2 , y_2) are the co-ordinates of the extremities of a focal chord of the parabola $y^2 = 4cx$, prove that $x_1 x_2 = c^2$ and $y_1 y_2 = - 4c^2$.

Q.23 If $y = \sqrt{5}x+k$, is a tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, what is the value of k.

Q.24 Find the equation of the tangents and normals to the following hyperbolas.

(i) $x^2-y^2=49$ at $(8,\sqrt{5})$ (ii) $49x^2-64y^2= 56$ at $(16,7\sqrt{3})$ (iii) $x^2-y^2=64$ at (10,6)

Q.25 Show that the eccentricities e_1 and e_2 of two conjugate hyperbolas satisfy the relation $e_1^2+e_2^2=e_1^2e_2^2$.

CHAPTER # 09 **VECTORS** **(Assignment #09)**

"Truth is ever to be found in the simplicity, and not in the multiplicity and confusion of things." — Isaac Newton

Multiple Choice Questions:

1. If $\vec{a} = P_1P_2$, when P_1(0,0,1) and P_2(0,4,4) then $|\vec{a}|$ is __________.
   a. 4 b. 5 c. 25 d. 9
2. If **a** and **b** any two vectors then $(a-b)\times(a+b) =$ ___________.
   a. a^2-b^2 b. 0 c. $a\times b$ d. $2(a\times b)$
3. If $\vec{a}.\vec{b}=0$, then the angle between the vectors $\vec{a}$ and $\vec{b}$ is __________.
   a. 0 b. $\frac{\pi}{2}$ c. $\frac{\pi}{3}$ d. π
4. $|\vec{a}|$ of a vector $\vec{a}$ when $\vec{a} = P_1P_2$, where P_1(0,0,1) and P_2(-3,1,2) is ________.
   a. $\sqrt{12}$ b. $\sqrt{10}$ c. $\sqrt{11}$ d. $\sqrt{13}$
5. If $i.[(2j-3k)\times(-2i+j+k)]$ is equal to __________.
   a. 5 b. 6 c. 4 d. none of these
6. If $\vec{F}=4i+j-3k$ and $\vec{r}=i+2j-3k$ then work done is ___________.
   a. 15 b. 16 c. 14 d. none of these
7. $k\times j =$ _________.
   a. $-i$ b. zero c. 1 d. i
8. If $\vec{a}=i, \vec{b}=j, \vec{c}=k$ then $\vec{a}.\vec{b}\times\vec{c} =$_________.
   a. 1 b. zero c. 2 d. 3
9. If the coordinates of two points P and Q are (2,1,-3) and (-1,2,4) respectively then vector $\overrightarrow{PQ}$ is _______.
   a. (-3,3,7) b. (2,4,-3) c. (3,3,3,) d. (-3,1,7)
10. If two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}.\vec{B}=0$ then the vectors are _________.
    a. Parallel b. perpendicular c. tangent d. none of these

Short-Answer Questions:

Q.1 Find scalar x, y, z such that: $x(3i-4k)+y(j-i+2k)+z(i-4k)=5i+4j-10k$

Q.2 Resolve the vectors a=(2,1,0) and b=(6,8,-6) in the direction of vectors P_1(1,-1,2), P_2(2,2,-1), P_3(3,7,-7).

Q.3 Two points P and Q have position vectors, with respect to an origin O, given by $3i-j+2k$ and $i-j-2k$ respectively. Calculate the length PQ, and show that the $\angle POQ$ is 90^0.

Q.4 Show that the position vector of the mid-point of the line AB where A and B have position vectors a and b respectively is $\frac{a+b}{2}$.

Q.5 P, Q, R are the points p, q and 2p-q respectively. M divides $\overline{PR}$ in 2:3 and N divides $\overline{QM}$ in 4:1, find the position vector of N.

Q.6 Find $m\angle B$ of the ΔABC, if A(4,1), B(-1,3) and C(-5,-2).

Q.7 Find cos $(\overrightarrow{AB},\overrightarrow{AC})$ in a triangle whose vertices are A(-2,0), B(4,3) and C(5,-1).

Q.8 Find cosC in a triangle ABC where A,B,C are the points A(-5,-4), B(-1,3), C(2,-3).

Q.9 Find sin (u,v) and also find a unit vector perpendicular to $u=i+2j+2k$ and $v=3i-2j-4k$.

Q.10 Find sin (a,b) for the vectors a, b of $a=2i+3j+4k$ and $b=i-j+k$.

Q.11 Find the unit vector perpendicular to the following pair of vectors u,v when $u=i=3j+3k, v=-3i+2k$.

Q.12 Find the scalar (area) of the triangle ABC where A, B, C are the points A(5,1,-2), B(-2-7-3), C(-4,-3-1).

Q.13 Find sin (a,b) and $\hat{u}$ when: (i) $\vec{a}=i-3j+4k, \vec{b}=-3i+3k$ (ii) $\vec{a}=i+2j+2k, \vec{b}=3i-2j+4k$
(iii) $\vec{a}=3i-6j-3k, \vec{b}=4i-3j+5k$ (iv) $\vec{a}=2i-3j+k, \vec{b}=i-2k$

Q.14 Find cos (a,b) when $\vec{a}=2i-2j+4k, \vec{b}=3i-6j-2k$.

Q.15 Find the volume of the parallelepiped with edges $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ where A, B, C are the points (0,1,1), (-2,1,3), (2,-2,0) respectively.

Q.16 Prove that: (i) $[\mathrm{a+b} \quad \mathrm{b+c} \quad \mathrm{c+a}] = 2[\mathrm{a} \ \mathrm{b} \ \mathrm{c}]$ (ii) $[\mathrm{e+f} \quad \mathrm{f+g} \quad \mathrm{g+e}] = 2[\mathrm{e}, \ \mathrm{f}, \ \mathrm{g}]$

Q.17 Evaluate the following scalar triple products.
(i) $[2i-3j, i+j-k, 3i-k]$ (ii) $[-2i+k, i, -i+2j+k]$

Q.18 Find the volume of the parallelepiped whose three adjacent edges are represented by the vectors:

(i) $\vec{a} = 2i - 3j + 4k, \vec{b} = i + 2j - k, \vec{c} = 3i - j + 2k$ (ii) $\vec{a} = i - 2j - 3k, \vec{b} = 2i + j - k, \vec{c} = i + 3j - 2k$

Q.19 Simplify: (i) $[a, 2b - 3c, -2a + b + c]$ (ii) $[-a - b - c, 2b + 3c, -4a + c]$

Q.20 Find the constant 'a' such that the following sets of vectors are coplanar.

(i) $2i - j + k, i + 2j - 3k, 3i + aj + 5k$ (ii) $i + 2j + k, aj - k, -2i + j$

Q.21 A particle at the corner of a cube, is acted upon by the magnitude 1, 2, 3 respectively, along the diagonals of the forces of the cube which meet at the particle. Find their resultant.

Q.22 A particle is acted on by the constant forces $4\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - \hat{k}$ and is displaced from the point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ to the point $5\hat{\imath} + 4\hat{\jmath} + \hat{k}$; Find the work done by the forces on the particle.

Q.23 Forces of magnitude 5, 3, 1 act on a particle in the directions of the vectors (6, 2, 3), (3, -2, 6), (2, -3, -6) respectively. The particle is displaced from, the point (2, -1, -3) to the point (5, -1, 1); find the work done by the forces.