XI Mathematics Formulae

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Chap#01 SETS

Number of elements in a set A = O(A), Set of Natural Numbers : N= {1,2,3,…}, Set of Whole Numbers :W= {0,1,2,3,…}, Set of Integers:Z= {o,±1, ±2, ±3,…}
Set of Odd Numbers :O = {±1, ±2, ±3,…}, Set of Positive Prime Numbers: P= {2,3,5,7,11,…}. Set of Even Numbers:E = {0, ±2, ±4,…},
Set of Rational Numbers :Q = {x│ x=$\frac{p}{q}$; p,q ϵ Z, q ≠ 0}, Set of Irrational Numbers :Q' = {x│ x ≠ $\frac{p}{q}$; p,q ϵ Z, q ≠ 0}, Set of Real Numbers:R = Q U Q' , Q U Q' =Ø = Null set

A is an improper subset of B ⟹A⊆B	A is proper subset of B ⟹ A⊂B	A and B are equal sets: A=B	A and B are equivalent sets i.e. O(A) = O(B) ⟹ A~B
union of A and B ⟹ AUB	intersection of A and B ⟹ A∩B	A and B are exhaustive sets if AUB = $\boldsymbol{U}$	A, B and C are exhaustive sets if AUBUC = $\boldsymbol{U}$
Difference of A and B=A-B	Complement of A = $\boldsymbol{U} - A$	Cartesian Product of A and B = A×B	Identity laws: AU Ø=A, A∩ Ø=Ø, AU $\boldsymbol{U}$=$\boldsymbol{U}$, A∩ $\boldsymbol{U}$ =$\boldsymbol{U}$
Complement laws: (A')'= A, Ø' = $\boldsymbol{U}$, A U A'= $\boldsymbol{U}$, A∩ A'= Ø, $\boldsymbol{U}$ ' = Ø		Demorgan's laws:(AUB)'=A' ∩B', (A∩ B)'=A' U B'	
Associative laws: (AUB)UC = AU(BUC), (A∩ B) ∩ C = A∩ (B∩C)		Commutative laws: AUB=BUA, A∩ B = B∩ A	
Distributive law of union of intersection: AU(B∩ C) = (AUB) ∩(AUC)		Distributive law of intersection over union: A ∩ (B UC) = (A ∩ B)U (A ∩ C)	
Distributive law of Cartesian product over union: A ×(BUC) = (A×B)U(A×C)		Distributive law of Cartesian product over intersection: A ×(B∩ C) = (A×B) ∩(A×C)	

Chap#02 REAL AND COMPLEX NUMBER SYSTEMS

Complex number = ℂ = ℝ × ℝ = {(a,b) |aϵ ℝ,bϵ ℝ}, $\mathbf{i} = \sqrt{-1}$, $\mathbf{i}^2 = -1$,(a,b) = a+b$\mathbf{i}$ =a+$\mathbf{i}$b, conjugate of (a,b) = (a,-b), conjugate of a+b $\mathbf{i}$= a-b$\mathbf{i}$

Equality: (a,b) = (c,d) ⟹a=c and b=d	Addition: (a,b)+(c,d)= (a+c, b+d)	Subtraction: (a,b) — (c,d) = (a-c , b-d)	Multiplication: (a,b)(c,d)= (ac-bd, ad+bc)
(a,b) ÷ (c,d) = $\left(\frac{ac+bd}{c^2+d^2}, \frac{bc-ad}{c^2+d^2}\right)$	Modulus of z = | z |	Conjugate coordinate of z=(x,y) = x+$\mathbf{i}$y are: x = $\frac{z+\bar{z}}{2}$, y = $\frac{z-\bar{z}}{2\mathbf{i}}$	
Additive inverse of (a,b) = (-a,-b) and additive inverse of a+b $\mathbf{i}$ = a-b $\mathbf{i}$		Multiplicative inverse of (a,b) = $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$	

Chap#03 EQUATIONS

If $ax^2+bx+c=0$, a≠ 0, is the Quadratic equation and x=$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is the quadratic formula,

$(a+b)^2 = a^2+2ab+b^2$	$a^2-b^2 = (a+b)(a-b)$	$(a+b)^3 = a^3+b^3+3ab(a+b)$	$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$	$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
$a^2+b^2 = (a+b)^2 - 2ab$	$a^3+b^3 = (a+b)^3 - 3ab(a+b)$	$a^3+b^3 = (a+b)(a^2-ab+b^2)$	$a^3-b^3 = (a-b)(a^2+ab+b^2)$	$a^3+b^3+c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
Cube roots of unity are 1, ω,ω^2 where $\omega\frac{-1+\mathbf{i}\sqrt{3}}{2}$&$\omega^2$ = $\frac{-1-\mathbf{i}\sqrt{3}}{2}$ also ω^3 = 1, 1+ ω +ω^2 = 0, $\mathbf{i}^2 = -1$				Nature of Roots: Discriminant = D = $b^2 - 4ac$
If D=0, then roots are equal		If D >0, i.e. +ve then roots are real and unequal	If D<0 i.e. –ve then roots are complex and unequal	
If D is a perfect square then roots are rational otherwise irrational		Relation b/w the roots α and β and the coefficient of a quadratic equation $ax^2+bx+c=0$ are as follows:		
Sum of roots = $\alpha+\beta = \frac{-b}{a}$		Product of roots =$\alpha\beta = \frac{c}{a}$	To form a quadratic equation when its roots are given: $x^2 - Sx + P = 0$ where S = Sum of roots and P = product of roots	

Chap#04 MATRICES

Square matrix: No. of rows = No. of columns	**Row matrix:** No. row = 1 e..g. [2,-9,4]		**Column matrix:** No. of column =1 e.g. $\begin{bmatrix}4\\-6\end{bmatrix}$
Rectangular matrix: No. of rows ≠ No. of columns e.g. $\begin{bmatrix}2 & 3 & -5\\0 & 8 & 4\end{bmatrix}$	**Upper triangular matrix:** $\begin{bmatrix}10 & 4 & -5\\0 & -2 & 9\\0 & 0 & 2\end{bmatrix}$		**Lower triangular matrix:** $\begin{bmatrix}10 & 0 & 0\\33 & -2 & 0\\4 & 2 & 12\end{bmatrix}$
Diagonal matrix: in which different elements are present only in diagonal e..g. $\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}$	**Idempotent matrix:** A^2= A		**Involutive matrix:** A^2=$\mathbf{I}$
Symmetric matrix: A^t= A	**Anti-symmetric matrix:** A^t= −A		**Orthogonal matrix:** A · A^t = I
Transpose of a matrix: A = $\begin{bmatrix}a & b\\c & d\end{bmatrix} \Rightarrow A^t = \begin{bmatrix}a & c\\b & d\end{bmatrix}$	Null matrix: $O_2 = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$, $O_3 = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ OR Zero matrix	Scalar matrix: A = $\begin{bmatrix}a & 0 & 0\\0 & a & 0\\0 & 0 & a\end{bmatrix}$	Unit matrix: $I_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, $I_3 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ OR Identity matrix
Order of a matrix: Rows× Column e.g. if a matrix has order 2×3 then it has 2 rows and 3 columns		Addition of matrices: Order of A should be equal to order of B	
Subtraction of matrices: Order of A should be equal to order of B		Multiplication of two matrices: No. of Column = No. of rows of B	

DETERMINANTS & INVERSE MATRICES

Expanding a determinant by R_1: $\begin{vmatrix}A & B & C\\D & E & F\\G & H & I\end{vmatrix}$ = A$\begin{vmatrix}E & F\\H & I\end{vmatrix}$ −B$\begin{vmatrix}D & F\\G & I\end{vmatrix}$ +C$\begin{vmatrix}D & E\\G & H\end{vmatrix}$ = A(EI - FH) – B (DI -FG) +C (DH - EG)

Expanding a determinant by C_1 : $\begin{vmatrix}A & B & C\\D & E & F\\G & H & I\end{vmatrix}$ = A $\begin{vmatrix}E & F\\H & I\end{vmatrix}$ −D$\begin{vmatrix}B & C\\H & I\end{vmatrix}$ + G$\begin{vmatrix}B & C\\E & F\end{vmatrix}$ = A(EI - FH) –D(BI - CH) + G(BF - CE)

Similarly a 4×4 determinant can be evaluated in the same manner: $\begin{vmatrix}A & B\,C & D\\E & F\,G & H\\I & J\,K & L\\L & M\,N & O\end{vmatrix} = A\begin{vmatrix}F & G & H\\J & K & L\\M & N & O\end{vmatrix} - B\begin{vmatrix}E & G & H\\I & K & L\\L & N & O\end{vmatrix} + C\begin{vmatrix}E & F & H\\I & J & L\\L & M & O\end{vmatrix} - D\begin{vmatrix}E & F & G\\I & J & K\\L & M & N\end{vmatrix}$

Properties of a determinant:1. $|A| = |A^t|$ 2. The interchanging of two rows or columns of a matrix A changes the sign of its determinant. 3. If any two rows or columns of a square matrix A are same the $|A| = 0$. 4. If all elements in a row or in a column of a square matrix A are zero then $|A| = 0$.

Singular matrix:$|A| = 0$ and **Non-singular matrix:**$|A| \neq 0$. Adjoint A = $\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}^t$, Multiplicative inverse $A^{-1} = \frac{Adj\,A}{|A|}$.

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Cramer's Rule: If a system of non-homogenous equations is a follow, $a_1x+a_2y+a_3z=a_4, b_1x+b_2y+b_3z=b_4, c_1x+c_2y+c_3z=c_4$ Now, $\lvert A\rvert=\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}$, $\lvert A_1\rvert=\begin{vmatrix}a_4&a_2&a_3\\b_4&b_2&b_3\\c_4&c_2&c_3\end{vmatrix}$, $\lvert A_2\rvert=\begin{vmatrix}a_1&a_4&a_3\\b_1&b_4&b_3\\c_1&c_4&c_3\end{vmatrix}$, $\lvert A_3\rvert=\begin{vmatrix}a_1&a_2&a_4\\b_1&b_2&b_4\\c_1&c_2&c_4\end{vmatrix}$ then $x=\frac{\lvert A_1\rvert}{\lvert A\rvert},\ y=\frac{\lvert A_2\rvert}{\lvert A\rvert},\ z=\frac{\lvert A_3\rvert}{\lvert A\rvert}$			
Matrix method: $\begin{bmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a_4\\b_4\\c_4\end{bmatrix}$ Now, AX = B ⟹X = A^{-1}B			

Chap#05 GROUPS

Binary Operation: satisfies the closure property w.r.t. ⋆ OR a⋆b ϵS ∀ a,b ϵ S. Properties of a Binary operation are as follow:
1.. Commutative: a⋆b = b⋆a 2. Associative: (a⋆b) ⋆ c = a⋆ (b ⋆c) 3. Identity element:a ⋆e = a 4. Inverse element: a ⋆ b = e

Groupoid:(S, ⋆)satisfies the closure property w.r.t. ⋆ OR a⋆b ϵS ∀ a,b ϵ S.	Semi Group OR Associative Groupoid satisfies 1. a⋆b ϵS 2. (a⋆b) ⋆ c = a⋆ (b ⋆c)
Group:satisfies 1.a⋆b ϵS 2.(a⋆b) ⋆ c = a⋆ (b ⋆c) 3. a ⋆e = a 4. a ⋆ b = e	Abelian Group: (G, ⋆)is a group which satisfies a⋆b = b⋆a

Multiplication OR Composite table: Let S={1,2,3,4} and we have to make composite table under usual multiplication. Then,

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Chap#06 SEQUENCES & SERIES

Arithmetic Progression OR Arithmetic Sequence: Arithmetic progression is a progression in which common difference is present so the question is that what is the common difference; second term minus first term, third term minus second term, every next term minus preceding term we get a constant answer, that constant answer is called c.d.
T_n =a + (n-1)d, where a=first term, n=number of terms, d=T_2-T_1 =common difference, T_n= l = n[th] term.

T_1= a = first term	T_2= a+d = second term	T_3= a + 2d = third term
Arithmetic Series: When we want to find the sum of an arithmetic sequence then it becomes Arithmetic series. Sum of n terms *is*: $S_n=\frac{n}{2}$\{ 2a + (n-1)d\} OR $S_n=\frac{n}{2}$\{a+l\}		
Arithmetic mean: Single mean b/w 'a' and 'b' is $\frac{a+b}{2}$	n A.M's b/w 'a' and 'b' are $A_1, A_2, A_3, \ldots A_n$ where: a=first term, b=last term, n=no. of means, d = $\frac{b-a}{n+1}$	
A_1 = a+d, A_2 = a+2d, A_3 = a + 3d . . . A_n = a+nd = b−d	Let three numbers in A.P. are (a-2d), a , (a+2d)	Let four numbers in A.P. are (a-3d), (a-d), (a+d),(a+3d)
Let five numbers in A.P. are (a-4d), (a-2d), a, (a+2d), (a+4d)	Let six numbers is A.P. are (a-5d), (a-3d), (a-d), (a+d), (a+3d), (a+5d)	

GeometricProgression OR Geometric Sequence: Geometric progression is a progression in which common ratio is present so the question is that what is the common ratio; second term divided by first term, third term divided second term, every next term divided by preceding term we get a constant answer, that constant answer is called c.r.
$T_n=ar^{n-1}$, where a = first term, n = number of terms, r= $\frac{T_2}{T_1}$ = common ratio, T_n= l = n[th] term.

T_1= a = first term	T_2= ar = second term	T_3= ar^2 = third term
Geometric Series: When we want to find the sum of a geometric sequence then it becomes geometric series. Sum of n terms is when: r <1,$S_n=\frac{a(1-r^n)}{1-r}$ OR$S_n=\frac{a-rl}{1-r}$ and when r >1,$S_n=\frac{a(r^n-1)}{r-1}$ OR $S_n=\frac{rl-a}{r-1}$, l = last term, S_n =Sum of n terms.		

Infinite Geometric Series: Is a geometric series in which number of terms are infinite, here always r < 1 and S denotes the sum of infinite terms.

S = $\frac{a}{1-r}$, r < 1	Geometric Means (G.M.): Single mean b/w 'a' and 'b' is $\sqrt{ab}$	Let 3 numbers in G.P. are$\frac{a}{r}$, a, ar
n G.M's b/w 'a' and 'b' are $G_1, G_2, G_3, \ldots G_n$ where: a=first term, b=last term, n=no. of means, r =$\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ = common ratio		Let 4 numbers in G.P. are $\frac{a}{r^3},\frac{a}{r}$, ar , ar^3
Let five numbers in G.P. are $\frac{a}{r^2}$, $\frac{a}{r}$, a , ar, ar^2		

HarmonicProgression OR Harmonic Sequence: Harmonic progression itself is not a progression but the reciprocal of A.P.

General Term: T_n =n[th] term, $T_n=\frac{ab}{b+(n-1)(a-b)}$ where n = no. of terms, a= first term, b= second term of the H.P.

Chap#07 PERMUTATIONS & COMBINATIONS

Counting: O(A)= number of elements in set A, If A={11,2,13,} then O(A)=3	**Counting principles:**If A and B are overlapping sets then, O(A)+O(B)=O(AUB)+O(A∩B)
If A and B are disjoint sets i.e. A∩B= Ø then O(AUB) = O(A) + O(B)	**Factorial notation:** Factorial n = n!, n!= n(n-1)! n!=n(n-1)(n-2)! n!=n(n-1)(n-2)(n-3)!
n!=n(n-1)(n-2)(n-3) . . . 3 . 2 . 1 0!=1 1!=1 2!=2 3!=6	No. of permutations: ${}^nP_r=\frac{n!}{(n-r)!}$ No. of Combinations: ${}^nC_r=\frac{n!}{r!(n-r)!}$
Group permutation:P = $\binom{n}{r,s,t}=\frac{n!}{r!.s!.t!}$ Circular permutation: P = (n-1)!	Permutation round a necklace: P = $\frac{(n-1)!}{2}$

Theorems: 1. $r!\binom{n}{r}={}^nP_r$ 2. ${}^nP_0$ =1 3.$\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$ 4.${}^nP_1=n$ 5.$\binom{n}{n-r}=\binom{n}{r}$ 6.${}^nP_n=n!$ 7.$\binom{n}{0}$ = 1 8.$\binom{n}{1}$ =n 9.$\binom{n}{n}$ = 1 10. (n+1)n! = (n+1)!

Division into Sections OR Parcels: 1. P = $\frac{(r+s)!}{r!.\ s!}$ 2. P = $\frac{(r_1+r_2+r_3+\ldots r_n)!}{r_1!.r_2!.\ldots r_n!}$ 3. P = $\frac{1}{n!}\left\{\frac{(nr)!}{(r!)^n}\right\}$

INTRODUCTION TO ROBABILITY

Sample Space:A set of all possible outcomes of an experiment is called the Sample Space and is denoted by S.
Event: Any subset of a sample space is called an event and is denoted by A.
Probability: P(A) = $\frac{O(A)}{O(S)}$ where P(A) = probability of event A, O(A) = no. of elements is set A, O(S) = No. of elements in set S.

Complementary events: P(A') = 1 – P(A)	Total probability: P(AUB)+P(A∩B)=P(A)+P(B)	If A and B are independent events i.e. A∩B= Ø, thenP(AUB) = P(A) + P(B)

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Chap#08 MATHEMATICAL INDUCTION

If P(n) is a proposition about a positive integer n such that:
1. P(n) is true for n = 1 2. P(n) is true for any positive integer n = k 3. P(n) is true for n = k+1, then P(n) is true for every positive integer.

$\sum n = 1 + 2 + 3 + \ldots n = \frac{n(n+1)}{2}$	$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$	$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{1}{4} n^2(n+1)^2$

BINOMIAL THEOREM

Binomial Theorem: $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-3)}{3!}a^{n-3}b^3 \ldots + b^n$		$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$	
Binomial Expansions: $(a+b)^2 = a^2 + 2ab + b^2$	$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$	$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2+4ab^3+b^4$	$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$
$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots$	$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots$	$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$	$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots$
General Term of $((A+B))^n$ is $T_{r+1} = n_{C_r}A^{n-r}B^r$ Note: r never being in ratio		**Middle terms:** 1. If n = even then $T_{r+1} = \left(\frac{n+2}{2}\right)$th term	
2. If n = odd then $T_{r+1} = \left(\frac{n+1}{2}\right)$th term and $T_{r+1} = \left(\frac{n+3}{2}\right)$th term		**Binomial Coefficients: Coefficient of** $T_1 = n_{C_0} = 1$	
Coefficient of $T_2 = n_{C_1} = n$	**Coefficient of** $T_3 = n_{C_2} = \frac{n(n-1)}{2!}$	**Coefficient of** $T_4 = n_{C_3} = \frac{n(n-1)(n-2)}{3!}$	**Coefficient of** $T_5 = n_{C_4} = \frac{n(n-1)(n-2)(n-3)}{4!}$
General Term for $(1+x)^n$: $T_{r+1} = \left\{\frac{n(n-1)(n-2)(n-3)\ldots(n-r+1)}{r!}\right\}x^r$		First approximation: $(1+x)^n$ = **1+nx**	
Second approximation: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$			

Chap#09 FUNDAMENTALS OF TRIGONOMETRY

Relation b/w Radian and Degree: π radians = 180^0 degrees, 1 Radian = $\frac{180}{\pi}$Degrees = 57.3^0, 1 Degree = $\frac{\pi}{180}$radians = 0.001745 Radians

Relation b/w Arc-length (S), Radius (r) and Central angle (θ) is S=rθ, Note: 1. 'θ' must be in Radian 2. The units of 'S' and 'r' should be same.	
The positive angle is measured in anti-clockwise order.	The negative angle is measured in clockwise order.

Angle in anti-clockwise direction 4-quadrants		Signs of the Trigonometric Functions in 4-quadrants		Angle in clockwise direction in 4-quadrants	
2^{nd} 90^0-180^0	0-90^0Ist	2^{nd}**Sinθ, cosecθ = +ve all others =-ve**	**All +ve Ist**	2^{nd} 180^0-270^0	270^0-360^0 Ist
3^{rd} 180^0-270^0	270^0-360^0 4th	3^{rd}**tanθ,cotθ= +ve all others =-ve**	**Cosθ, secθ = +ve, 4^{th} all others =-ve**	3^{rd} 90^0-180^0	0-90^0 4^{th}

Trigonometric Identities: $\sin^2\theta + \cos^2\theta = 1$	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$	Sin θ = y, cos θ = x	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
Reciprocal Trigonometric Ratios: cosec $\theta = \frac{1}{\sin\theta}$	$\sin\theta = \frac{1}{\csc\theta}$	Cot = $\frac{1}{\tan\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cos\theta = \frac{1}{\sec\theta}$

Table for Trigonometric ratios:

Angle θ	**0**	$\frac{\pi}{6} = 30^0$	$\frac{\pi}{4} = 45^0$	$\frac{\pi}{3} = 60^0$	$\frac{\pi}{2} = 90^0$
Sinθ	0	$\frac{1}{2} = 0.5$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{\sqrt{3}}{2}$	1
Cosθ	1	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{2}$	0
Tanθ	0	$\frac{1}{\sqrt{3}} = 0.577$	1	$\sqrt{3}$	∞
Cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Chap#10 TRIGONOMETRIC IDENTITIES

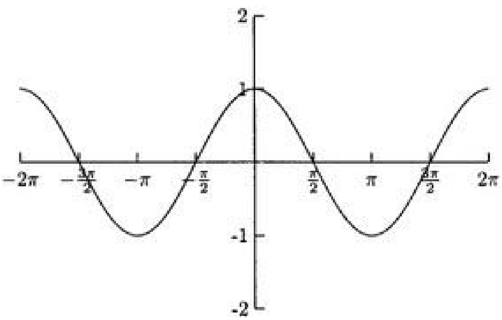
Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$		Mid-point formula: $x = \frac{x_1+x_2}{2}$ and $y = \frac{y_1+y_2}{2}$	
Fundamental Law: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$	$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\cos 2\theta = 2\cos^2\theta - 1$, $\cos 2\theta = 1 - 2\sin^2\theta$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$	$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
$\tan = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$	**Product to Sum and Difference formulae:** 1. $\sin\alpha.\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta) + \sin(\alpha-\beta)\}$ 2. $\cos\alpha.\sin\beta = \frac{1}{2}\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$ 3. $\cos\alpha.\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$ 4. $\sin\alpha.\sin\beta = -\frac{1}{2}\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\}$		
Sum and Difference to Product formulae: 1. $\sin U + \sin V = 2\sin\frac{U+V}{2}\cos\frac{U-V}{2}$ 2. $\sin U - \sin V = 2\cos\frac{U+V}{2}\sin\frac{U-V}{2}$ 3. $\cos U + \cos V = 2\cos\frac{U+V}{2}\cos\frac{U-V}{2}$			
4. $\cos U - \cos V = -2\sin\frac{U+V}{2}\sin\frac{U-V}{2}$	$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$	$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$	

$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$	$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$	$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

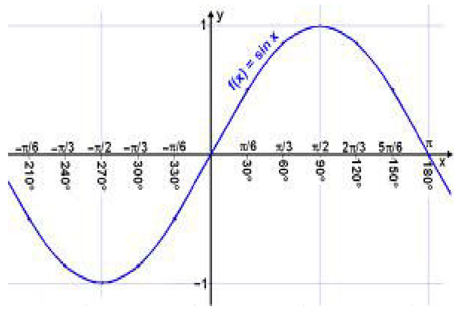
Chap#11 GRAPHS OF TRIGONOMETRIC FUNCTIONS

Period of sin θ is 2π ; $\sin(\theta + 2\pi) = \sin\theta$	Period of cos θ is 2π ; $\cos(\theta + 2\pi) = \cos\theta$	Period of tan θ is π ; $\tan(\theta + \pi) = \tan\theta$

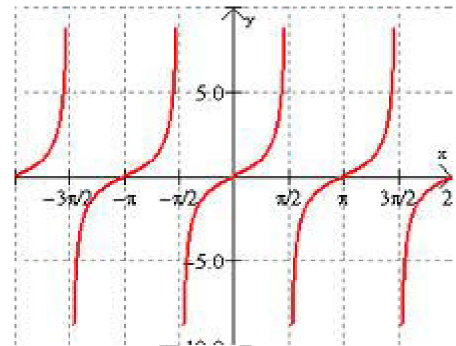
Graph of cosθ



Graph of Sineθ



Graph of tan θ



Chap#12 SOLUTIONS OF TRIANGLES

Law of Sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ OR $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$	Law of Cosines: $\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$ OR $a^2 = b^2 + c^2 - 2ab.\cos\alpha$
$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$ OR $b^2 = a^2 + c^2 - 2ac.\cos\beta$	$\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$ OR $c^2 = a^2 + b^2 - 2ab.\cos\gamma$

Law of tangents: $\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$	$\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$	$\frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

Half angle formulae: $\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$	$\sin\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$	$\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$	$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$	$\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$
$\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$	$\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$	$\tan\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$	$\tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$	$\tan\frac{\alpha}{2} = \frac{r}{s-a}$; $\tan\frac{\beta}{2} = \frac{r}{s-b}$; $\tan\frac{\gamma}{2} = \frac{r}{s-c}$
s =semi perimeter , $s = \frac{a+b+c}{2}$	$r^2 = \frac{(s-a)(s-b)(s-c)}{s}$	Area of triangle ABC: $\Delta = \frac{1}{2}ab\sin\gamma$	$\Delta = \frac{1}{2}bc\sin\beta$	

$\Delta = \frac{1}{2}ca\sin\gamma$	$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$	$\Delta = \frac{1}{2}a^2\frac{\sin\beta.\sin\gamma}{\sin\alpha}$	$\Delta = \frac{1}{2}b^2\frac{\sin\alpha.\sin\gamma}{\sin\beta}$	$\Delta = \frac{1}{2}c^2\frac{\sin\alpha.\sin\beta}{\sin\gamma}$

R= Radius of Circum circle, $R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$ OR $R = \frac{abc}{4\Delta}$	r = Radius of Inscribed circle (Incircle) $r = \frac{\Delta}{s}$

r_1 , r_2 , r_3 = radius of escribed circle, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

Chap#13 INVERSE TRIGONOMETRIC FUNCTIONS & TRIGONOMETRIC EQUATIONS

Subject Specialist:
Faizan Ahmed
"Merging man and Math"

Tan A $\pm$ tan B =tan $\left(\frac{A \pm B}{1 \mp AB}\right)$

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