XI MATHEMATICS

SOLUTION OF PAPER 2013

Including:

- Grading of marks
- Showing Marks of each step
- Step wise solution

Note: See the Solution of every Question very carefully and note the common mistakes which are done normally by the Students so that you can get 100 marks in Mathematics.

PROVIDED BY

BOARD OF INTERMEDIATE EDUCATION, KARACHI

BOARD OF INTERMEDIATE EDUCATION, KARACHI
INTERMEDIATE EXAMINATION, 2013 (ANNUAL)

Date: 23.05.2013
9:50 a.m. to 12:30 p.m.

MATHEMATICS PAPER – I

(Science Pre-Engineering & Science General Groups)

Max. Marks: 80
Time: 2 hours 40 minutes

INSTRUCTION: The use of calculator is allowed. All notations are used in their usual meanings. Graph paper will be supplied on demand.

SECTION 'B'
(SHORT-ANSWER QUESTIONS) (50 Marks)

ALGEBRA (35 Marks)

NOTE: Attempt any Seven part questions from this Section. All questions carry equal marks.

2. i) Prove that the roots of the equation $x^2-2x\left(m+\frac{1}{m}\right)+3=0$ are real, $\forall m\in\mathbb{R}$.

ii) If α,β are the roots of the equation $2x^2+3x+4=0$. Form the equation whose roots are α^2 and β^2.

iii) Solve the following:

$$x^2+y^2=25$$
$$(4x-3y)(x-y-5)=0$$

OR

Solve: $\sqrt{\frac{x+16}{x}}+\sqrt{\frac{x}{x+16}}=\frac{25}{12}$

iv) Let $S=\{A,B,C,D\}$ where $A=\{a\}, B=\{a,b\}, C=\{a,b,c\}$ and $D=\varphi$. Construct composition table to show that $\cup$ and $\cap$ are binary operations on S.

v) In how many ways can 3 books of Mathematics, 2 books of Physics and 2 books of Chemistry be placed on a shelf so that the books on the same subject always remain together?

vi) Identify the series as binomial expansion and find its sum $1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}.\frac{1}{2^2}+\frac{2.5.8}{3.6.9}.\frac{1}{2^3}+\ldots\ldots$

vii) Prove the given proposition by the principle of mathematical induction:

$$1^2+2^2+3^2+\ldots\ldots\ldots\ldots+n^2=\frac{n(n+1)(2n+1)}{6},\ \forall n\in\mathbb{N}$$

viii) Find x, y, z and v so that $\begin{bmatrix}4 & x+y\\ z+v & 3\end{bmatrix}=3\begin{bmatrix}x & y\\ z & v\end{bmatrix}+\begin{bmatrix}x & 6\\ -1 & 2v\end{bmatrix}$

ix) If c be a quantity, so small that c^3 may be neglected in comparison with l^3, prove that $\sqrt{\frac{l}{l+c}}+\sqrt{\frac{l}{l-c}}=2+\frac{3c^2}{4l^2}$.

x) Let $S=\{1,\omega,\omega^2\}$ where ω, being complex cube root of unity, construct multiplication table with respect to $(\cdot)$ and show that:

a) $(\cdot)$ is binary operation in S   b) 1 is the identity element in S

TRIGONOMETRY (15 Marks)

NOTE: Attempt any Three part questions from this Section. All questions carry equal marks.

ii) Prove any Two of the following:

a) $\dfrac{1+\sec\theta}{1-\sec\theta}=\dfrac{\tan\theta+\sin\theta}{\sin\theta-\tan\theta}$, $(\cos\theta\neq 1)$

b) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

c) $\sin 5\theta - \sin 3\theta + \sin 2\theta = 4\sin\theta\cos\dfrac{3\theta}{2}\cos\dfrac{5\theta}{2}$

iii) A piece of plastic strip 1 meter long is bent to form an isosceles triangle with 95° as its largest angle. Find the length of the sides.

iv) A belt 24.75 metres long passes around a 3.5 cm diameter pulley. As the belt makes three complete revolutions in a minute, how many radians does the wheel turn in one second?

v) Prove that: $\mathrm{Tan}^{-1}\dfrac{1}{13}+\mathrm{Tan}^{-1}\dfrac{1}{4}=\mathrm{Tan}^{-1}\dfrac{1}{3}$.

SECTION 'C'
(DETAILED-ANSWER QUESTIONS) (30 Marks)

NOTE: Attempt any Two questions from this Section. All questions carry equal marks.

4. a) Find the inverse of $A=\begin{bmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & 2 & -3\end{bmatrix}$ by adjoint method. **8**

OR

By using the properties of determinants express the following determinant in factorized form $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ bc & ac & ab\end{vmatrix}$.

b) If α and β are the roots of $pt^2+qt+q=0$, $p\neq 0$, $q\neq 0$ prove that $\sqrt{\dfrac{q}{p}}+\sqrt{\dfrac{\alpha}{\beta}}+\sqrt{\dfrac{\beta}{\alpha}}=0$. **7**

5. a) Which term of the sequence 18, 12, 8 .... is $\dfrac{512}{729}$? **3**

b) Prove that a, b, c are in A.P., G.P. or H.P. according as $\dfrac{a-b}{b-c}=\dfrac{a}{a}$ *or* $\dfrac{a}{b}$ *or* $\dfrac{a}{c}$ **5**

OR

Insert four Harmonic means between 12 and $\dfrac{48}{5}$.

c) Write in the simplified form the term involving x^{10} in the expansion of $\left(x^2-\dfrac{1}{x^3}\right)^{10}$. **7**

6. a) Prove that in any triangle ABC, $r_1r_2r_3=rs^2$ **4**

b) Prove that in any triangle ABC, $r=\dfrac{\Delta}{s}$. **5**

OR

Draw the graph of $\cos 2\theta$ where $-180^\circ\leq\theta\leq 180^\circ$.

c) Find the general solution of $\sin\theta+\cos\theta=1$ **6**

BOARD OF INTERMEDIATE EDUCATION, KARACHI

Annual Examination 2013

Solution and instructions of Sections "B" & "C"

Mathematics Paper-IScience Pre-Eng.&General

Q.2(i) $x^2 - 2x\left(m + \frac{1}{m}\right) + 3 = 0$

Here $a = 1; b = -2\left(m + \frac{1}{m}\right)$ and $c = 3$

For real roots of quadratic eq.

$\because D = b^2 - 4ac > 0$

$\Rightarrow D = \left\{-2\left(m + \frac{1}{m}\right)\right\}^2 - 4(1)(3)$ ———————— +2

$\Rightarrow D = 4\left\{\left(m + \frac{1}{m}\right)^2 - 3\right\}$

$\Rightarrow D = 4\left(m^2 + 2 + \frac{1}{m^2} - 3\right)$

$\Rightarrow D = 4\left(m^2 + \frac{1}{m^2} - 1\right)$

$\Rightarrow D = 4\left\{\left(m - \frac{1}{m}\right)^2 + 1\right\} > 0$ ———————— +2

Which is positive $\forall m \in \mathbb{R}$ .Hence the roots of the given equation are real. ——— +1

Proved.

Q.2(ii) $2x^2 - 3x + 4 = 0$

Here $a = 2; b = -3$ and $c = 4$

$\because$ α and β are the roots of the given equation

$\therefore \alpha + \beta = -\frac{b}{a} = +\frac{3}{2}$

And $\alpha\beta = \frac{c}{a} = \frac{4}{2} = 2$ ———————— +1

Roots of required eq are α^2 and β^2 then

Sum of the roots $= \alpha^2 + \beta^2$

Sum of the roots $= (\alpha + \beta)^2 - 2\alpha\beta$

Sum of the roots $= \frac{9}{4} - 4$

$\boxed{\text{Sum of the roots} = -\frac{7}{4}}$ —————— + 2

Product of the roots $= \alpha^2\beta^2$

Product of the roots $=(\alpha\beta)^2$

$\boxed{\text{Product of the roots} = 4}$ —————— + 1

∴ required equation is given by

$x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$

$x^2 - \left(-\frac{7}{4}\right)x + (4) = 0$

$4x^2 + 7x + 16 = 0$ —————— + 1

Q.2(iii) $\quad x^2 + y^2 = 25 \longrightarrow (1)$

$(4x - 3y)(x - y - 5) = 0 \longrightarrow (2)$

From Eq(2)

Either	or
$4x - 3y = 0$	$x - y - 5 = 0$
$\Rightarrow 4x = 3y$	$\Rightarrow x = y + 5 \longrightarrow (4)$
$\Rightarrow x = \frac{3}{4}y \longrightarrow (3)$	Putting in Eq(1)
Putting in Eq(1)	$\Rightarrow (y+5)^2 + y^2 = 25$
$\Rightarrow \left(\frac{3}{4}y\right)^2 + y^2 = 25$	$\Rightarrow y^2 + 10y + 25 + y^2 = 25$
$\Rightarrow \frac{9y^2}{16} + y^2 = 25$	$\Rightarrow 2y^2 + 10y = 0$
$\Rightarrow \frac{9y^2+16y^2}{16} = 25$	$\Rightarrow 2y(y+5) = 0$
$\Rightarrow 25y^2 = 25(16)$	Either Or
$\Rightarrow y^2 = 16$	$\Rightarrow 2y = 0 \quad y + 5 = 0$
$\Rightarrow y = \pm 4$	$\Rightarrow y = 0 \quad \Rightarrow y = -5$ —————— +1
When $y = 4$	when $y = 0$
Eq(3) $\Rightarrow x = 3$	Eq(4) $\Rightarrow x = 5$
When $y = -4$	when $y = -5$

$Eq(3) \Rightarrow x = -3$ $\qquad$ $Eq(4) \Rightarrow x = 0$ ———— + 1

$\therefore S.S. = \{(3,4), (-3,-4), (5,0), (0,-5)\}$ *Ans.* ———— + 1

OR

Q.2(iii) $\sqrt{\frac{x+16}{x}} + \sqrt{\frac{x}{x+16}} = \frac{25}{12} \longrightarrow (1)$

Let $\sqrt{\frac{x+16}{x}} = y \longrightarrow (2)$

Then Eq(1) becomes

$y + \frac{1}{y} = \frac{25}{12}$

$\Rightarrow \frac{y^2+1}{y} = \frac{25}{12}$

$\Rightarrow 12y^2 + 12 = 25y$

$\Rightarrow 12y^2 - 25y + 12 = 0$

Factorizing

$\Rightarrow 12y^2 - 16y - 9y + 12 = 0$

$\Rightarrow 4y(3y-4) - 3(3y-4) = 0$

$\Rightarrow (3y-4)(4y-3) = 0$

Either $3y - 4 = 0$ $\qquad$ or $4y - 3 = 0$

$\Rightarrow y = \frac{4}{3}$ $\qquad$ $\Rightarrow y = \frac{3}{4}$ ———— + 3

$Eq(2) \Rightarrow \sqrt{\frac{x+16}{x}} = \frac{4}{3}$ $\qquad$ $Eq(2) \Rightarrow \sqrt{\frac{x+16}{x}} = \frac{3}{4}$

Squaring both sides $\qquad$ Squaring both sides

$\Rightarrow \frac{x+16}{x} = \frac{16}{9}$ $\qquad$ $\Rightarrow \frac{x+16}{x} = \frac{9}{16}$

$\Rightarrow 9x + 144 = 16x$ $\qquad$ $\Rightarrow 16x + 256 = 9x$

$\Rightarrow 9x - 16x = -144$ $\qquad$ $\Rightarrow 16x - 9x = -256$

$\Rightarrow -7x = -144$ $\qquad$ $\Rightarrow 7x = -256$

$\Rightarrow x = \frac{144}{7}$ $\qquad$ $\Rightarrow x = \frac{-256}{7}$

$\therefore S.S. = \left\{\frac{144}{7}, \frac{-256}{7}\right\}$ *Ans.* ———— + 2

Q.2 (iv)

$\cup$	A	B	C	D
A	A	B	C	A
B	B	B	C	B
C	C	C	C	C
D	A	B	C	D

+2

Since all the elements in the above table belong to S.

$\therefore \cup$ is a binary operation on S. Proved. ———— +1

$\cap$	A	B	C	D
A	A	A	A	D
B	A	B	B	D
C	A	B	C	D
D	D	D	D	D

Since all the elements in the above table belong to S.

$\therefore \cap$ is a binary operation on S. Proved. ———— +2

Q.2(v) There are 3 books of Mathematics, 2 books of Physics and 2 books of Chemistry on a shelf. We have to arrange these books on a shelf so that the books on the same subject remain together, then we form a unit containing 3 books of Mathematics, another unit comprising 2 books of Physics and one unit consisting 2 books of Chemistry.

These 3 units can be arranged in 3! ways. ———— +2

3 Mathematics books can be arranged within themselves in 3! Ways, 2 Physics books can be arranged within themselves in 2! ways and 2 Chemistry books can be arranged within themselves in 2! ways ———— +2

Total Number of ways $= 3!\cdot 3!\cdot 2!\cdot 2!$

$= 6\cdot 6\cdot 2\cdot 2$

$= 144$ *Ans.* ———— +1

Q.2(vi) Let $1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2\cdot 5}{3\cdot 6}\cdot\frac{1}{2^2}+\frac{2\cdot 5\cdot 8}{3\cdot 6\cdot 9}\cdot\frac{1}{2^3}+\cdots=(1+x)^n \longrightarrow (1)$

$$1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2\cdot 5}{3\cdot 6}\cdot\frac{1}{2^2}+\frac{2\cdot 5\cdot 8}{3\cdot 6\cdot 9}\cdot\frac{1}{2^3}+\cdots=1+nx+\frac{n(n-1)}{2!}x^2+\cdots$$ ———— +1

On comparing

$nx = \frac{1}{3} \longrightarrow (2)$ $\qquad$ $\frac{n(n-1)}{2!}x^2 = \frac{5}{36} \longrightarrow (3)$

Solving Eq(2) and Eq(3)

$\boxed{n = -\frac{2}{3}}$ and $\boxed{x = -\frac{1}{2}}$ ———————— +2

Putting these values in Eq(1)

$$1 + \frac{2}{3}\cdot\frac{1}{2} + \frac{2\cdot5}{3\cdot6}\cdot\frac{1}{2^2} + \frac{2\cdot5\cdot8}{3\cdot6\cdot9}\cdot\frac{1}{2^3} + \cdots = \left(1 - \frac{1}{2}\right)^{-2/3} \quad Ans. \text{ ———— } +1$$

$$= \left(\frac{1}{2}\right)^{-2/3}$$

$$= (2)^{2/3}$$

$$= 4^{1/3}$$

$$= \sqrt[3]{4} \quad Ans. \text{ ———————— } +1$$

+2

Q.2(vii) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$; $\forall n \in N$

Condition (1): For n=1

$$1^2 = \frac{1(1+1)\{2(1)+1\}}{6}$$

$$\Rightarrow 1 = \frac{1\,(2)\,(3)}{6}$$

$$\Rightarrow 1 = 1$$

$\therefore$ Condition (1) is satisfied. ———————— +1

Condition (2):

Let the given proposition be true for $n = k$

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6} \text{ ———————— } +1$$

Adding $(k+1)^{th}$ term i.e. $(k+1)^2$ on both sides

$$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left\{\frac{k(2k+1)}{6} + (k+1)\right\}$$

$$= (k+1)\left\{\frac{k(2k+1)+6(k+1)}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2+k+6k+6}{6}\right\}$$

$$= (k+1)\left(\frac{2k^2+7k+6}{6}\right)$$

$$= (k+1)\left(\frac{2k^2+4k+3k+6}{6}\right)$$

$$= (k+1)\left\{\frac{2k(k+2)+3(k+2)}{6}\right\}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(\overline{k+1}+1)(2k+2+1)}{6}$$

$$= \frac{(k+1)(\overline{k+1}+1)\{2(k+1)+1\}}{6}$$

Which is true for $n = k+1$ whenever it is true for $n = k$.

∴ Condition (2) is also satisfied . ———————— +2

Hence by the Principle of Mathematical induction the given proposition is true $\forall n \in N$. Proved. ———————— +1

Q.2(viii) $\begin{bmatrix} 4 & x+y \\ z+v & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & v \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2v \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & x+y \\ z+v & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3v \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2v \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & x+y \\ z+v & 3 \end{bmatrix} = \begin{bmatrix} 4x & 3y+6 \\ 3z-1 & 5v \end{bmatrix}$ ———————— +2

By equality of matrices

$4 = 4x \longrightarrow (1)$

$x + y = 3y + 6 \longrightarrow (2)$

$z + v = 3z - 1 \longrightarrow (3)$

$3 = 5v \longrightarrow (4)$ ———————— +1

Eq(1) $\Rightarrow \boxed{x = 1}$

Eq(2) $\Rightarrow 1 + y = 3y + 6$

$\Rightarrow 2y = -5$

$\Rightarrow \boxed{y = -\frac{5}{2}}$

Eq(4) $\Rightarrow \boxed{v = \frac{3}{5}}$

Eq(3) $\Rightarrow z + \frac{3}{5} = 3z - 1$

$\Rightarrow 2z = \frac{3}{5} + 1$

$\Rightarrow 2z = \frac{8}{5}$

$\Rightarrow \boxed{z = \frac{4}{5}}$

$\therefore x = 1;\ y = -\frac{5}{2};\ z = \frac{4}{5}$ *and* $v = \frac{3}{5}$ *Ans.* ———————— +2

Q.2(ix) L.H.S.$= \sqrt{\frac{l}{l+c}} + \sqrt{\frac{l}{l-c}}$

$$= \frac{\sqrt{l}}{\sqrt{l+c}} + \frac{\sqrt{l}}{\sqrt{l-c}}$$

$$= \frac{l^{1/2}}{l^{1/2}\left(1+\frac{c}{l}\right)^{1/2}} + \frac{l^{1/2}}{l^{1/2}\left(1-\frac{c}{l}\right)^{1/2}}$$

$$= \left(1+\frac{c}{l}\right)^{-1/2} + \left(1-\frac{c}{l}\right)^{-1/2}$$

Expanding by Binomial Theorem

$$= \left\{1 + \left(-\frac{1}{2}\right)\frac{c}{l} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{c}{l}\right)^2 \textit{ by neglecting } c^3\textit{, and higher powers of } c\right\}$$

$$+ \left\{1 + \left(-\frac{1}{2}\right)\left(-\frac{c}{l}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{c}{l}\right)^2 \textit{ by neglecting } c^3\textit{, and higher powers of } c\right\}$$

$$= \left\{1 - \frac{c}{2l} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{c^2}{l^2}\right\} + \left\{1 + \frac{c}{2l} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{c^2}{l^2}\right\}$$ ———————— +3

$$= 2 + \frac{3c^2}{8l^2} + \frac{3c^2}{8l^2}$$

$$= 2 + \frac{6c^2}{8l^2}$$

$= 2 + \frac{3c^2}{4l^2}$ Proved. ———————— +2

Q.2(x) $S = \{1, \omega, \omega^2\}$

Table:

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	$\omega^3 = 1$	$\omega^4 = \omega$

———————— +2

(a) $\forall a, b \in S$

$a \cdot b \in S$

Since all the elements in the above table belong to set S

$\therefore$ Multiplication is a binary operation in S. ———————— +1

(b) $1.\omega = \omega \in S$

$1.\omega^2 = \omega^2 \in S$

$1 \cdot 1 = 1 \in S$

$\therefore$ 1 is the identity element in S Proved. ———————— +1

TRIGONOMETRY

Q.3 (i) since $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$ ———————— +2

$\because \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\pm\sqrt{1-\cos^2\theta}}{\cos\theta}$

$\because \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\pm\sqrt{1-\cos^2\theta}}$ ———————— +2

$\because \text{Sec}\,\theta = \frac{1}{\cos\theta}$

$\because \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\pm\sqrt{1-\cos^2\theta}}$ ———————— +1

OR

Q.3(i) $\cot\theta = 3$

$\because$ $\cot\theta$ and $\sin\theta$ are both positive. Therefore $\rho(\theta)$ lies in the first quadrant. ———————— +1

Let $\rho(\theta) = (x,y)$ lies on a unit circle, where $x = \cos\theta, y = \sin\theta$ and $x^2 + y^2 = 1$

$\because \cot\theta = \frac{x}{y} = 3$

$\Rightarrow x = 3y$ ———→ (1) ———————— +1

$\because x^2 + y^2 = 1$

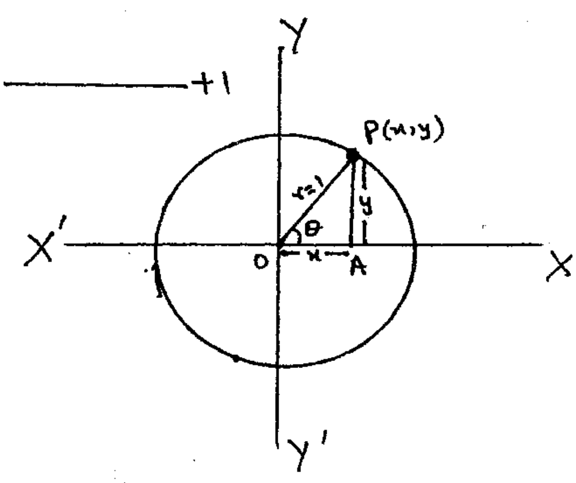
$\Rightarrow (3y)^2 + y^2 = 1$

$\Rightarrow 9y^2 + y^2 = 1$

$\Rightarrow 10y^2 = 1$

$\Rightarrow y^2 = \frac{1}{10}$

$\Rightarrow y = \pm\sqrt{\frac{1}{10}}$



$\Rightarrow y = \sin\theta = \frac{1}{\sqrt{10}}$ ——————— +1

$\text{Eq(1)} \Rightarrow x = 3\left(\sqrt{\frac{1}{10}}\right)$

$\Rightarrow x = \cos\theta = \frac{3}{\sqrt{10}}$

$\because \tan\theta = \frac{y}{x} = \frac{1/\sqrt{10}}{3/\sqrt{10}} = \frac{1}{3}$

$\because \text{Sec}\,\theta = \frac{1}{x} = \frac{\sqrt{10}}{3}$

$\because \text{cosec}\,\theta = \frac{1}{y} = \sqrt{10}$

Hence the remaining trigonometric functions are

$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\text{cosec}\,\theta$
$\frac{1}{\sqrt{10}}$	$\frac{3}{\sqrt{10}}$	$\frac{1}{3}$	$\frac{\sqrt{10}}{3}$	$\sqrt{10}$

——+2

Q.3(ii) (a) $\frac{1+\sec\theta}{1-\sec\theta} = \frac{\tan\theta+\sin\theta}{\sin\theta-\tan\theta}$ (3+2)

$R.H.S. = \frac{\tan\theta+\sin\theta}{\sin\theta-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\sin\theta-\frac{\sin\theta}{\cos\theta}}$ $\because \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\cancel{\sin\theta}\left(\frac{1}{\cos\theta}+1\right)}{\cancel{\sin\theta}\left(1-\frac{1}{\cos\theta}\right)}$

$= \frac{\frac{1}{\cos\theta}+1}{1-\frac{1}{\cos\theta}}$

$= \frac{1+\sec\theta}{1-\sec\theta}$

$= L.H.S.$

L.H.S.=R.H.S. Proved.

Q.3(ii)(b) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

L.H.S.= $\cos 3\theta$

$= \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta - 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$= R.H.S.$

L.H.S.=R.H.S. Proved.

Q.3(ii)(c) $\sin 5\theta - \sin 3\theta + \sin 2\theta = 4\sin\theta\cos\frac{3\theta}{2}\cos\frac{5\theta}{2}$

L.H.S.$= \sin 5\theta - \sin 3\theta + \sin 2\theta$

$= \left(2\cos\frac{5\theta+3\theta}{2}\cdot\sin\frac{5\theta-3\theta}{2}\right) + \sin 2\theta$

$= 2\cos 4\theta \cdot \sin\theta + 2\sin\theta\cos\theta$

$= 2\sin\theta(\cos 4\theta + \cos\theta)$

$= 2\sin\theta\left(2\cos\frac{4\theta+\theta}{2}\cdot\cos\frac{4\theta-\theta}{2}\right)$

$= 4\sin\theta\cos\frac{5\theta}{2}\cos\frac{3\theta}{2}$

$= 4\sin\theta\cos\frac{3\theta}{2}\cos\frac{5\theta}{2}$

$= R.H.S.$

L.H.S.=R.H.S. Proved.

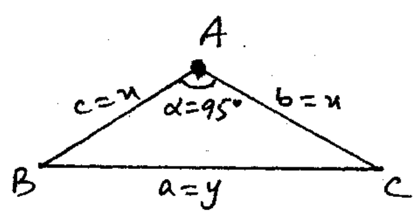
Q.3(iii) Let x be the length of the two equal sidesand y be the length of the unequal side of an isosceles triangle ABC with $\alpha = 95^\circ$ as its largest angle.
Using Law of Cosine

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\Rightarrow y^2 = x^2 + x^2 - 2\cdot x\cdot x\cdot\cos 95^\circ$

$\Rightarrow y^2 = 2x^2 - 2x^2\cos 95^\circ$

$\Rightarrow y^2 = 2x^2 - 2x^2(-0.087156)$



$\Rightarrow y^2 = 2x^2 + 0.17431x^2$

$\Rightarrow y^2 = 2.17431x^2$

$\Rightarrow y = 1.47455x \dashrightarrow (1)$ ———— +2

Also

$a + b + c = 1\, metre$

$\Rightarrow y + x + x = 1$

$\Rightarrow y = 1 - 2x \dashrightarrow (2)$ ———— +1

Solving Eq(1) and Eq(2) simultaneously, we have

$x = 0.2878$ m and $y = 0.4244$ m

Hence the lengths of the three sides of the isosceles triangle are 0.2878 metre, 0.2878 metre and 0.4244 metre. Ans. ———— +2

Q.3 (iv) Length of the belt $= 24.75$ metres.

Diameter of wheel of the pulley $= 3.5$ cm.

$$\because r = \frac{diameter}{2}$$

$$\Rightarrow r = \frac{3.5}{2} = 1.75 \text{ cm.}$$

$$\Rightarrow r = \frac{1.75}{100} = 0.0175 \text{ metre}$$ ———— +1

As the belt makes three complete revolutions in one minute

$s =$ Distance covered by the belt in one minute $= 3(24.75)$

$= 74.25$ m.

$\Rightarrow s =$ Distance covered by the belt in one second $= \frac{74.25}{60}$ m.

$\Rightarrow s = 1.2375$ metres ———— +2

$\because s = r\theta$

$\Rightarrow \theta = \frac{s}{r}$

$\Rightarrow \theta = \frac{1.2375}{0.0175}$

$\Rightarrow \theta = 70.714$ radians/sec. Ans. ———— +2

Q.3(v) $\tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{1}{3}$

L.H.S.$= \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{4}$

Let $\tan^{-1}\frac{1}{13} = \alpha$ and $\tan^{-1}\frac{1}{4} = \beta$

$\Rightarrow \tan\alpha = \frac{1}{13}$ and $\Rightarrow \tan\beta = \frac{1}{4}$ ———————— +1

$\therefore$ L.H.S.$= \alpha + \beta$

$= \tan^{-1}\{\tan(\alpha+\beta)\}$

$= \tan^{-1}\left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)$ ———————— +1

$= \tan^{-1}\left(\frac{\frac{1}{13}+\frac{1}{4}}{1-\frac{1}{13}\frac{1}{4}}\right)$

$= \tan^{-1}\left(\frac{\frac{4+13}{52}}{\frac{52-1}{52}}\right)$

$= \tan^{-1}\left(\frac{17}{51}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

$= R.H.S$

$\therefore$ L.H.S.=R.H.S. Proved. ———————— +3

SECTION 'C'

DETAILED-ANSWER QUESTIONS (30 marks)

Q.4(a) $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$

$\because A^{-1} = \frac{Adj.A}{|A|} ----\rightarrow (1)$

$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{vmatrix}$

Expanding by 1^{st} row

$|A| = 2\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 5 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 0 & 2 \\ 5 & 2 \end{vmatrix}$

$\Rightarrow |A| = 2(-6-2) - 1(0-5) - 1(0-10)$

$\Rightarrow |A| = 2(-8) - 1(-5) - 1(-10)$

$\Rightarrow |A| = -16 + 5 + 10$ ∵ Matrix A is non-singular.

$\Rightarrow |A| = -1 \neq 0$ ∴ Its inverse exists. ———— +1

$\because A_{ij} = (-1)^{i+j} \cdot M_{ij}$

$\therefore A_{11} = (-1)^{1+1} \cdot M_{11}$

$A_{11} = (-1)^2 \begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix}$

$A_{11} = 1(-6-2)$

$\Rightarrow A_{11} = -8$

$A_{12} = (-1)^3(0-5) = 5$

$A_{13} = (-1)^4(0-10) = -10$

$A_{21} = (-1)^3(-3+2) = 1$

$A_{22} = (-1)^4(-6+5) = -1$

$A_{23} = (-1)^5(4-5) = 1$

$A_{31} = (-1)^4(1+2) = 3$

$A_{32} = (-1)^5(2-0) = -2$

$A_{33} = (-1)^6(4-0) = 4$ ———— +3

$\because AdjA = [A_{ij}]^t$

$\because AdjA = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$

$\Rightarrow AdjA = \begin{bmatrix} -8 & 5 & -10 \\ 1 & -1 & 1 \\ 3 & -2 & 4 \end{bmatrix}^t$

$\Rightarrow AdjA = \begin{bmatrix} -8 & 1 & 3 \\ 5 & -1 & -2 \\ -10 & 1 & 4 \end{bmatrix}$ ———— +2

$\text{Eq}(1) \Rightarrow A^{-1} = \dfrac{\begin{bmatrix} -8 & 1 & 3 \\ 5 & -1 & -2 \\ -10 & 1 & 4 \end{bmatrix}}{-1}$

$\Rightarrow A^{-1} = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ Ans. ———— +2

OR (NOTE: Without using Properties, award zero marks.)

Let $|A| = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{vmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 1-1 & 1-1 \\ a & b-a & c-a \\ bc & ac-bc & ab-bc \end{vmatrix} \quad \begin{matrix} C_1(-1)+C_2 \\ C_1(-1)+C_3 \end{matrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 0 & 0 \\ a & -(a-b) & c-a \\ bc & c(a-b) & -b(c-a) \end{vmatrix}$$

$$\Rightarrow |A| = (a-b)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & -1 & 1 \\ bc & c & -b \end{vmatrix} \quad \textit{Taking out } (a-b) \textit{ common from } C_2 \textit{ and } (c-a) \textit{ common from } C_3$$

$$\Rightarrow |A| = (a-b)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & -1 & 1-1 \\ bc & c & c-b \end{vmatrix} \quad C_2(1)+C_3$$

$$\Rightarrow |A| = (a-b)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ bc & c & c-b \end{vmatrix}$$

$$\Rightarrow |A| = (a-b)(c-a)(c-b)\begin{vmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ bc & c & 1 \end{vmatrix} \quad \textit{Taking out } (c-b) \textit{ common from } C_3 \quad +1$$

Now expanding by 1^{st} row

$$\Rightarrow |A| = (a-b)(c-a)(c-b)\left\{1\begin{vmatrix} -1 & 0 \\ c & 1 \end{vmatrix}\right\}$$

$$\Rightarrow |A| = (a-b)(c-a)(c-b)(-1-0)$$

$$\Rightarrow |A| = (a-b)(b-c)(c-a)$$ Ans. +2

Q.4 (b) $pt^2+qt+q=0$

Here $a=p; b=c=q$

If α and β are the roots, then

$\alpha+\beta = -\frac{b}{a} = -\frac{q}{p}$

$\alpha\beta = \frac{c}{a} = \frac{q}{p}$ +2

Now,

$$L.H.S. = \sqrt{\frac{q}{p}} + \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$$

$$= \sqrt{\frac{q}{p}} + \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}}$$

$$= \sqrt{\frac{q}{p}} + \frac{\alpha+\beta}{\sqrt{\alpha}\sqrt{\beta}}$$

$$= \sqrt{\frac{q}{p}} + \frac{(-{}^{q}/_{p})}{\sqrt{{}^{q}/_{p}}} \quad \text{———————— } +3$$

$$= \sqrt{\frac{q}{p}} - \frac{{}^{q}/_{p}}{\sqrt{{}^{q}/_{p}}}$$

$$= \sqrt{\frac{q}{p}} - \sqrt{\frac{q}{p}}$$

$$= 0$$

$= R.H.S.$ Proved. ———————— +2

Q.5(a) $18, 12, 8, \cdots$ is $\frac{512}{729}$

Here $a = 18$ Common ratio $r = \frac{12}{18} = \frac{2}{3}$

$T_n = \frac{512}{729}$ $n = ?$

$\because T_n = a \cdot r^{n-1}$ ———————— +1

$$\Rightarrow \frac{512}{729} = 18 \cdot \left(\frac{2}{3}\right)^{n-1}$$

$$\Rightarrow \frac{512}{729 \times 18} = \left(\frac{2}{3}\right)^{n-1}$$

$$\Rightarrow \frac{256}{6561} = \left(\frac{2}{3}\right)^{n-1}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{8} = \left(\frac{2}{3}\right)^{n-1}$$

$$\Rightarrow n - 1 = 8$$

$$\Rightarrow n = 9$$

$\therefore \frac{512}{729}$ is the 9^{th} term of the G.P. ———————— +2

Q.5(b) $\because \frac{a-b}{b-c} = \frac{a}{a}$

$\Rightarrow \frac{a-b}{b-c} = 1$

$\Rightarrow a - b = b - c$

$\Rightarrow a + c = b + b$

$\Rightarrow 2b = a + c$

$\Rightarrow b = \frac{a+c}{2}$

$\Rightarrow$ b is the arithmetic mean between a and c. ———————— +2

$\Rightarrow a, b, c$ are in $A.P.$ Proved.

$\because \frac{a-b}{b-c} = \frac{a}{b}$

$\Rightarrow b(a - b) = a(b - c)$

$\Rightarrow ab - b^2 = ab - ac$

$\Rightarrow b^2 = ac$

$\Rightarrow b = \pm\sqrt{ac}$ ———————— +1

$\Rightarrow$ b is the geometric mean between a and c.

$\Rightarrow a, b, c$ are in $G.P.$ Proved.

$\because \frac{a-b}{b-c} = \frac{a}{c}$

$\Rightarrow c(a - b) = a(b - c)$

$\Rightarrow ac - bc = ab - ac$

$\Rightarrow ac + ac = ab + bc$

$\Rightarrow b(a + c) = 2ac$

$\Rightarrow b = \frac{2ac}{a+c}$

$\Rightarrow$ b is the harmonic mean between a and c.

$\Rightarrow a, b, c$ are in $H.P.$ Proved. ———————— +2

OR

Q.5(b) Let H_1, H_2, H_3 *and* H_4 be the four harmonic means between 12 and $\frac{48}{5}$, then

$12, H_1, H_2, H_3, H_3, \frac{48}{5}$ are in H.P.

$\frac{1}{12}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{5}{48}$ are in A.P. ———— +1

$\because$ Common difference $d = \frac{b-a}{n+1}$

Here $a = \frac{1}{12}$; $b = \frac{5}{48}$ and $n = 4$

$\because d = \frac{\frac{5}{48} - \frac{1}{12}}{4+1}$

$\Rightarrow d = \frac{\frac{5-4}{48}}{5}$

$\Rightarrow d = \frac{1}{240}$ ———— +1

Now $\frac{1}{H_1} = a + d = \frac{1}{12} + \frac{1}{240} = \frac{21}{240} = \frac{7}{80} \quad \Rightarrow H_1 = \frac{80}{7}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{21}{240} + \frac{1}{240} = \frac{22}{240} = \frac{11}{120} \quad \Rightarrow H_2 = \frac{120}{11}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{22}{240} + \frac{1}{240} = \frac{23}{240} \quad \Rightarrow H_3 = \frac{240}{23}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = \frac{23}{240} + \frac{1}{240} = \frac{24}{240} = \frac{1}{10} \quad \Rightarrow H_4 = 10$

$\therefore$ The four Harmonic means between 12 and $\frac{48}{5}$ are $\frac{80}{7}, \frac{120}{11}, \frac{240}{23}$ and 10. ———— +3

Ans.

Q.5(c) $\left(x^2 - \frac{1}{x^3}\right)^{10}$

The general term is given by

$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

Here $\quad a = x^2; b = -\frac{1}{x^3}; n = 10$ ———— +1

$\therefore T_{r+1} = {}^{10}C_r \cdot (x^2)^{10-r} \cdot \left(\frac{-1}{x^3}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r \cdot x^{20-2r} \cdot \frac{(-1)^r}{x^{3r}}$

$\Rightarrow T_{r+1} = {}^{10}C_r \cdot (-1)^r \cdot x^{20-2r} \cdot x^{-3r}$

$\Rightarrow T_{r+1} = {}^{10}C_r \cdot (-1)^r \cdot x^{20-5r}$ ———— +3

For the term involving x^{10},

we take $20 - 5r = 10$

$$\Rightarrow -5r = -10$$

$$\Rightarrow r = 2$$ ———————— +1

$$\therefore T_{2+1} = {}^{10}C_2 \cdot (-1)^2 \cdot x^{20-52}$$

$$\Rightarrow T_3 = \frac{10!}{2!(10-2)!} \cdot 1 \cdot x^{10}$$

$$\Rightarrow T_3 = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 8!} x^{10}$$

$$\Rightarrow T_3 = 45\, x^{10}$$ Ans. ———————— +2

Q.6 (a) $r_1 r_2 r_3 = rs^2$

$L.H.S. = r_1 r_2 r_3$

$$= \frac{\Delta}{s-a} \frac{\Delta}{s-b} \frac{\Delta}{s-c}$$ ———————— +1

$$= \frac{\Delta^2 \cdot \Delta}{(s-a)(s-b)(s-c)}$$

Multiplying and dividing by s

$$= \frac{\Delta^2 \cdot \Delta \cdot s}{s(s-a)(s-b)(s-c)}$$ ———————— +1

$$= \frac{\Delta^2 \cdot \Delta \cdot s}{\Delta^2}$$

$$= \Delta \cdot s$$

$$= (rs)s$$

$$= rs^2$$

$$= R.H.S$$

$\therefore L.H.S. = R.H.S.$ Proved. ———————— +2

Q.6(b)

Proof: Consider a triangle ABC. Draw the bisectors of angles A, B and C so that they meet at the incentre I. Draw the perpendiculars $\overline{ID}, \overline{IE}$ *and* $\overline{IF}$ on $\overline{AB}, \overline{BC}$ *and* $\overline{AC}$ respectively and the draw the incircle with radius r.

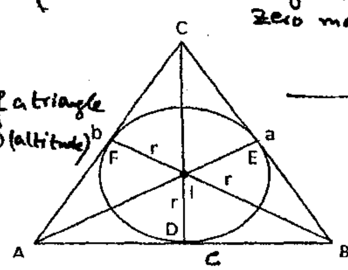
$$\therefore r = m\,\overline{IF} = m\,\overline{IE} = m\,\overline{ID}$$

(NOTE: Without Fig. Award zero marks.)

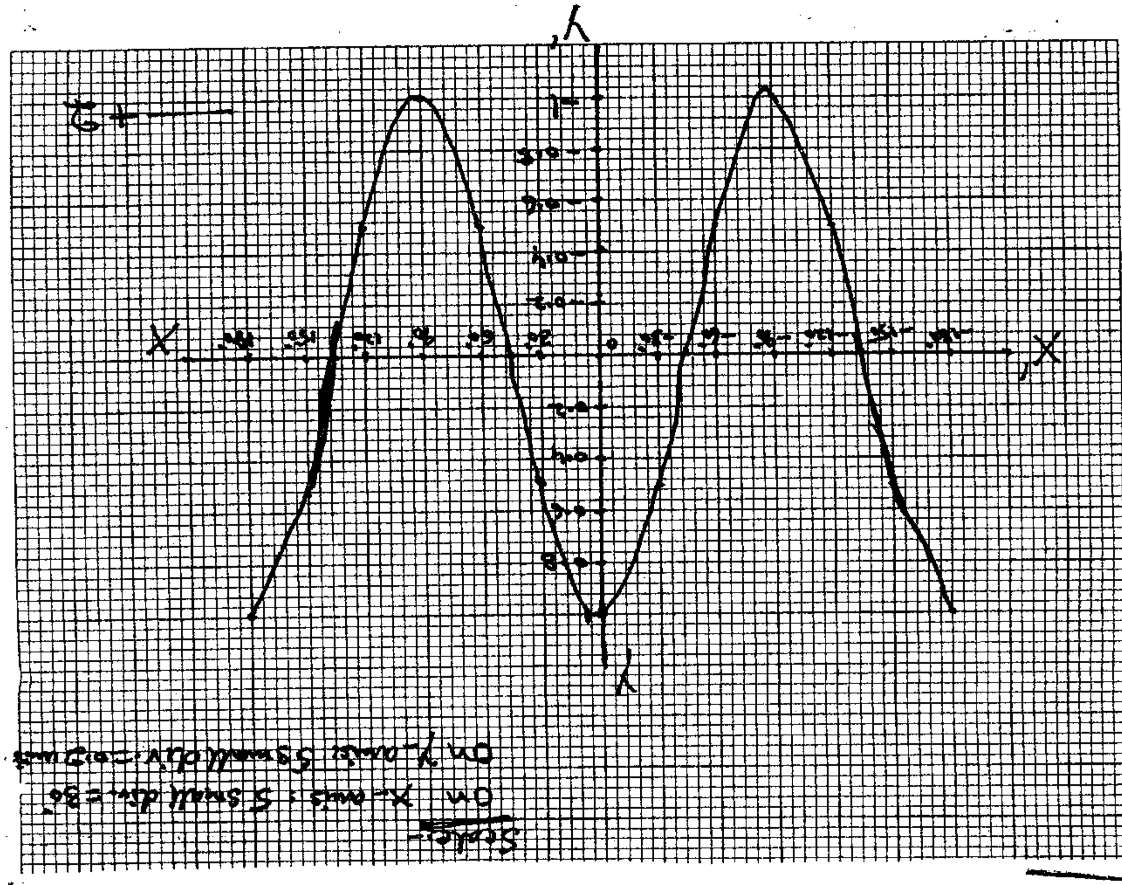
Now $\Delta ABC = \Delta\, ABI + \Delta\, BCI + \Delta\, CAI$

$$\Delta ABC = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br$$

$\because$ Area of a triangle $= \frac{1}{2}$(base)(altitude) ——— +1



——— +2


Scale:
On X-axis: 5 small div = 36°
On Y-axis: 5 small div = 0.2 unit

$$\Delta ABC = \frac{1}{2} r(a+b+c)$$

$$\Delta ABC = \frac{1}{2} r(2s)$$

$$\Delta ABC = rs$$

or $\Delta = rs$

$\boxed{r = \frac{\Delta}{s}}$ Hence proved. ———————— +2

OR

Q.6(b) Graph of $\cos 2\theta$ where $-180^\circ \le \theta \le 180^\circ$

(NOTE: Without Table, award zero marks.)

θ	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°
2θ	-360°	-300°	-240°	-180°	-120°	-60°	0°	60°	120°	180°	240°	300°	360°
$\cos 2\theta$	1	0.5	-0.5	-1	-0.5	0.5	1	0.5	-0.5	-1	-0.5	0.5	1

→ +2

Scale:- On x-axis 5small divisions = 30°

On y-axis 5small divisions = 0.2 unit ———— +1

Q.6(c) $\sin\theta + \cos\theta = 1$

5 8a

$\Rightarrow \cos\theta = 1 - \sin\theta$

Squaring both sides

$\Rightarrow \cos^2\theta = (1 - \sin\theta)^2$

$\Rightarrow \cos^2\theta = 1 - 2\sin\theta + \sin^2\theta$

$\Rightarrow 1 - \sin^2\theta = 1 - 2\sin\theta + \sin^2\theta$

$\Rightarrow 2\sin^2\theta - 2\sin\theta = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 1) = 0$

Either	Or
$2\sin\theta = 0$	$\sin\theta - 1 = 0$
$\Rightarrow \sin\theta = 0$	$\Rightarrow \sin\theta = 1$
$\Rightarrow \theta = \sin^{-1} 0$	$\Rightarrow \theta = \sin^{-1} 1$
$\Rightarrow \theta = 0,\ \theta = \pi$	$\Rightarrow \theta = \frac{\pi}{2}$ ———— +3

VERIFICATION:-

Put $\theta = 0$ in Eq.(1)	Put $\theta = \frac{\pi}{2}$ in Eq.(1)
$\sin 0 + \cos 0 = 1$	$\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$
$0 + 1 = 1$	$1 + 0 = 1$
$1 = 1$	$1 = 1$
Verified	Verified
Put $\theta = \pi$ in Eq.(1)	$\therefore \theta = \frac{\pi}{2}$
$\sin\pi + \cos\pi = 1$	$\theta = \frac{\pi}{2} + 2n\pi$
$0 - 1 = 1$	
$-1 \neq 1$	
Not verified	———— +2

$\therefore \theta = 0$

$\theta = 2n\pi$ $\quad \therefore S.S. = \{2n\pi\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\},\ n \in Z$ ———— +1